

# 渗透水压下岩体多裂纹相互作用的计算

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**摘要:**根据断裂力学理论,建立渗透水压下多裂隙岩体的力学模型,运用叠加原理,推导远场应力和裂隙水压力共同作用下的多裂纹尖端应力强度因子的计算公式,并在此基础上探讨了近置多裂纹间的相互影响。计算结果表明,如果裂纹之间间距较小,裂纹的相互作用对裂纹尖端处的应力场影响较大,根据裂纹的相对几何位置和裂纹倾角变化,进行不同叠加后的应力场可产生2种效应,应力强度影响加强区与应力强度影响减弱区,当相邻裂纹尖端处于加强区时,该裂纹的应力强度因子将会变大,反之则变小。考虑裂纹间的相互作用与裂纹间的相对距离有关,随着裂纹间距的增大,计算所得出的应力强度因子值都趋于一个定值,该值为不考虑相邻裂纹的影响,计算单一裂纹所得到的应力强度因子值,即当裂纹间距>裂纹长度时,可以忽略邻近裂纹的相互作用对裂纹尖端应力场的影响。同时,裂隙渗透压的存在,显著影响裂纹尖端的应力场,各条裂纹尖端 $K_I$ 随着渗透压的增大而增加(使 $-K_I$ 逐渐减小),这表明渗透压抵消了一部分裂纹表面正应力。

**关键词:**渗透压;多裂纹;相互作用;应力强度因子

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## A New Method for Analysis of Interaction of Rock Mass with Multiple Cracks under Seepage Pressure

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**Abstract:** According to the theory of fracture mechanics, the mechanical model of rock-mass with multi-cracks under the action of seepage pressure was set up. Based on the principle of superposition, the formula for calculating stress intensity factor (SIF) was built. The stress intensity factor of cracks in different conditions, such as different crack distance, different crack angle and different seepage pressure, were discussed. It is suggested that, if distance between two cracks is short, the interaction of cracks has significant influence on stress field of crack tip. According to the change of relative crack location and crack angle, the different superposed stress can produce two effects, stress field strengthen and stress field weaken. The stress intensity factor would be increased when the crack tip is in the stress strengthen area; otherwise this value would be decreased. The interaction of adjacent cracks is related to the distance between two cracks. As the crack distance increases, the SIF is close to a fixed value which is calculated without considering the interaction between cracks. Therefore, when the crack spacing is larger than crack length, the interaction between adjacent cracks can be ignored. The results also show that the seepage pressure significantly affects the stress field near crack tip, and the stress intensity factor  $K_I$  increases along with the increase of seepage pressure ( $-K_I$  decreases).

**Key words:** seepage pressure; multiple cracks; interaction; stress intensity factors

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地下岩体常以多裂纹的面貌出现,由于裂纹尖端应力分布的奇异性,容易引发裂纹尖端材料局部失效而导致宏观破坏,岩石断裂力学的研究主要集中在对均质材料中特殊单一裂纹或者规律性分布裂纹断裂力学行为的探讨,进而对复杂多裂纹岩体的力学行为进行研究。多裂纹相互作用分析,作为岩体材料细观破损分析的关键问题,近几十年来获得了广泛的关注,发展了多种分析方法<sup>[1-10]</sup>。基于应力叠加技术,Kachanov 提出了一种计算多裂纹相互作用下应力强度因子的简单方法,并进一步用来估算多裂纹体等效弹性参数及分析裂纹体的应力、应变场<sup>[6-7]</sup>。Li 等<sup>[10]</sup>,李银平<sup>[11]</sup>,H. Qing<sup>[12]</sup>等对 Kachanov 法提出了改进,提高了计算精度。Basista<sup>[8]</sup>、李银平<sup>[13]</sup>等将该方法推广到压缩荷载下准脆性物体中多个压剪裂纹相互作用的问题。实际工程中的岩体赋存于一定的地应力和地下水压力环境中,地下水与岩体相互作用。处于渗透压下的多裂隙岩体,其力学行为将发生改变。地应力在裂隙面的法向分量一半使结构面受到压缩并使深埋岩体中的裂隙闭合,而地下水压力作用于裂隙面的法向,力图使裂隙面张开。无渗透水压时裂纹尖端的应力为压应力,在渗透水压下可能转化为拉应力,裂纹尖端的复杂应力状态直接影响着裂纹尖端的应力强度因子值。关于渗透压下的岩桥贯通规律,学者已作了一些理论及试验研究<sup>[14-16]</sup>,但对于应力场和渗透压共同作用下岩体中多裂纹相互作用的研究甚少。

本文根据断裂力学理论,建立无限大板含多裂纹的力学模型,根据叠加原理,推导远场应力和裂隙水压力共同作用下的多裂纹应力强度因子,并在此基础上研究渗透水压下多裂纹的相互作用,为渗透压条件下的多裂隙岩体断裂失稳的研究提供初步理论依据。

## 1 水-岩力学作用下多裂纹应力强度因子计算

考虑裂隙渗透水压的情况如图 1 所示,无限平面  $x$ - $y$  内含有  $N$  条裂纹,裂纹长度为  $2a_i$  ( $i = 1, \dots, N$ ),与  $x$  轴的夹角为  $\alpha_i$  ( $i = 1, \dots, N$ ),受远场双轴压应力  $\sigma^\infty = -(\sigma_1^\infty, \sigma_3^\infty)$ ,裂隙水渗透压力为  $p^w$ ,垂直于裂纹表面。

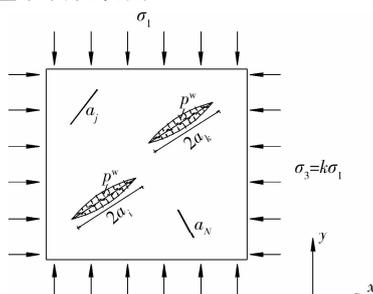


图 1 渗透水压下多裂纹计算模型

对任意裂纹  $i$ ,根据任意斜面上应力分量变换关系,可知作用于裂纹面上的法向及切向应力分别为<sup>[16]</sup>

1) 裂纹未闭合情况:

$$\sigma_i = \left[ \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\alpha_i \right] - p_i^w \quad (1)$$

$$\tau_i = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\alpha_i$$

2) 裂纹闭合情况:

$$\sigma_i = \left[ \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\alpha_i \right] - p_i^w$$

$$\tau_i = \left| \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\alpha_i \right| - \sigma_i H(\sigma_i) \tan \varphi_i - C_i \quad (2)$$

$$H(\sigma_i) = \begin{cases} 1 & (\sigma_i > 0) \\ 0 & (\sigma_i \leq 0) \end{cases}$$

其中,  $\tan \varphi_i$  为摩擦系数,  $C_i$  为内聚力。

将此问题分解为  $N$  个子问题,每个子问题中只含有一条裂纹,而无穷远处应力为零,每条裂纹面上受法向及切向应力作用,是初始面力和由于其他  $N-1$  条裂纹的存在而引起的附加面力两者之和。如图 2 所示,例如:第  $k$  裂纹面上所受的法向及切向应力为  $p_k(s_k)$  和  $q_k(s_k)$ 。该问题又可以分解为  $N$  个子问题,每个子问题中只有一条单裂纹,且裂纹表面应力情况未知,例如:单独考虑第  $k$  裂纹,其裂纹面上的应力为  $P_k(s_k)$  和  $Q_k(s_k)$ 。

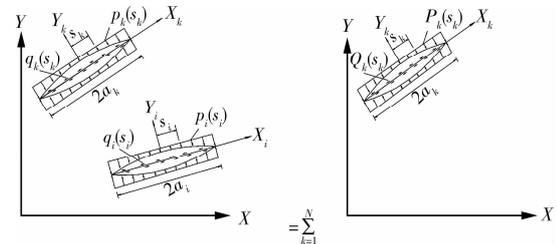


图 2 多裂纹叠加示意图

根据叠加原理,得到 Fredholm 积分方程<sup>[5]</sup>

$$P_k(s_k) + \sum_{l=1, l \neq k}^N \int_{-a_l}^{a_l} P_l(s_l) f_{m, lk}(s_l, s_k) ds_l +$$

$$\sum_{l=1}^N \int_{-a_l}^{a_l} Q_l(s_l) f_{m, lk}(s_l, s_k) ds_l = p_k(s_k),$$

$$(-a_k < s_k < a_k) \quad k = 1, 2, \dots, N$$

$$Q_k(s_k) + \sum_{l=1, l \neq k}^N \int_{-a_l}^{a_l} P_l(s_l) f_{n, lk}(s_l, s_k) ds_l +$$

$$\sum_{l=1}^N \int_{-a_l}^{a_l} Q_l(s_l) f_{n, lk}(s_l, s_k) ds_l = q_k(s_k),$$

$$(-a_k < s_k < a_k) \quad k = 1, 2, \dots, N \quad (3)$$

式中,  $f_m, f_n, f_m, f_n$  为相互作用系数,表达式见附录 A。假设 Fredholm 积分方程在离散点  $M$  处能获得满意的计算结果,则由式(3)即可得到  $N$  条裂纹的  $2 \times N \times M$  个方程,再利用 Chebyshev 数值迭代规则即可得到

$$P_k(s_i) + \sum_{l=1, l \neq k}^N \sum_{j=1}^M P_l(t_j) f_{m, lk}(t_j, s_i) \delta_j +$$

$$\sum_{l=1, l \neq k}^N \sum_{j=1}^M Q_l(t_j) f_{m, lk}(t_j, s_i) \delta_j = p_k(s_i),$$

$$i, j = 1, 2, \dots, M$$

$$Q_k(s_i) + \sum_{l=1, l \neq k}^N \sum_{j=1}^M P_l(t_j) f_{m, lk}(t_j, s_i) \delta_j +$$

$$\sum_{l=1, l \neq k}^N \sum_{j=1}^M Q_l(t_j) f_{u, lk}(t_j, s_i) \delta_j = q_k(s_i),$$

$$i, j = 1, 2, \dots, M \quad (4)$$

式中,

$$s_i = a \cos \frac{(2i-1)\pi}{2M} \quad i = 1, 2, \dots, M$$

$$t_j = a \cos \frac{(2j-1)\pi}{2M} \quad (5)$$

$$\delta_j = \frac{\pi a}{M} \sin \frac{(2j-1)\pi}{2M} \quad j = 1, 2, \dots, M \quad (5)$$

通过分析,将边界条件带入式(1)、(2),即可以通过 MATLAB 编程计算得出  $P_k, Q_k$ ,从而由下式得到水-岩力学作用下各裂纹尖端的应力强度因子

$$K_I(\pm a_k) = \frac{1}{\sqrt{\pi a_k}} \int_{-a_k}^{a_k} P_k(s_k) \sqrt{\frac{a_k + s_k}{a_k - s_k}} d s_k \quad (6)$$

$$K_{II}(\pm a_k) = \frac{1}{\sqrt{\pi a_k}} \int_{-a_k}^{a_k} Q_k(s_k) \sqrt{\frac{a_k + s_k}{a_k - s_k}} d s_k$$

## 2 算例分析

### 2.1 2 条裂纹算例分析

无限板中含 2 条裂纹,取  $2a=20$  mm,裂纹之间垂直距离  $2d=2$  mm,平行距离  $2b=4$  mm,取  $\sigma_1 = 10$  MPa,  $\sigma_3 = 4$  MPa,裂隙水压力  $p=2$  MPa,如图 3 所示。

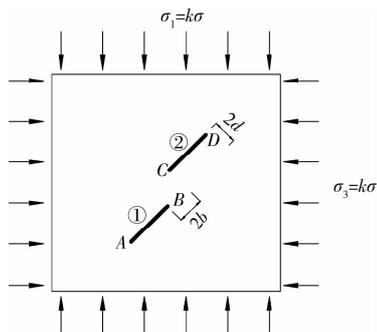


图 3 2 条裂纹计算模型

根据式(3)得到各裂纹的相关方程并代入式(4)得

$$P_1(s_i) + \sum_{j=1}^M P_2(t_j) f_{m, 21}(t_j, s_i) \delta_j +$$

$$\sum_{j=1}^M Q_2(t_j) f_{m, 21}(t_j, s_i) \delta_j = \sigma_1 \quad i = 1, 2, \dots, M$$

$$Q_1(s_i) + \sum_{j=1}^M Q_2(t_j) f_{u, 21}(t_j, s_i) \delta_j +$$

$$\sum_{j=1}^M Q_2(t_j) f_{u, 21}(t_j, s_i) \delta_j = \tau_1 \quad i = 1, 2, \dots, M \quad (7)$$

$$P_2(t_i) + \sum_{j=1}^M P_1(s_j) f_{m, 12}(s_j, t_i) \delta_j +$$

$$\sum_{j=1}^M Q_1(s_j) f_{m, 12}(s_j, t_i) \delta_j = \sigma_2 \quad i = 1, 2, \dots, M$$

$$Q_2(t_i) + \sum_{j=1}^M P_1(s_j) f_{u, 12}(s_j, t_i) \delta_j +$$

$$\sum_{j=1}^M Q_1(s_j) f_{u, 12}(s_j, t_i) \delta_j = \tau_2 \quad i = 1, 2, \dots, M \quad (8)$$

求解以上方程组即可求得 2 条裂纹相互影响下的  $P_1(s_i), Q_1(s_i), P_2(t_i), Q_2(t_i)$ ,代入式(6),计算得到不同裂纹倾角下裂纹尖端各点的应力强度因子值。如果不考虑裂纹之间的相互影响,根据断裂力学理论,将式(1)代入下式,即可得到任一裂尖端应力强度因子为

$$K_I^0 = -\sigma_i \sqrt{\pi a} \quad K_{II}^0 = \tau_i \sqrt{\pi a} \quad (9)$$

将虑裂纹间相互影响计算得到的结果与上式进行对比,得到无量纲应力强度因子值,见图 4。

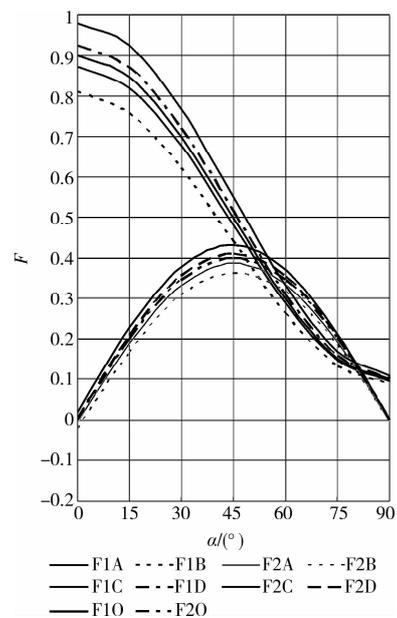


图 4 2 条裂纹 F1A, F1B..., F2D 值

### 2.2 3 条裂纹算例分析

无限板中含 3 条裂纹,取  $2a=20$  mm,①②裂纹间  $2b=10$  mm,①③裂纹间  $2d=4$  mm,取  $\sigma_1 = 10$  MPa,  $\sigma_3 = 4$  MPa,裂隙水压力  $p=2$  MPa,如图 5 所示。

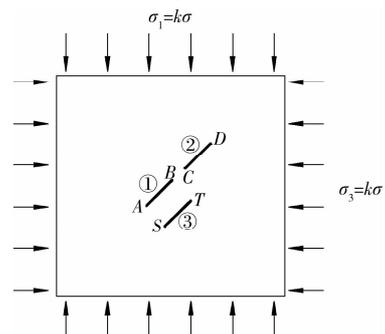


图 5 3 条裂纹计算模型

根据式(3)得到各裂纹的相关方程并代入式(4)得

$$P_1(s_i) + \left[ \sum_{j=1}^M P_2(t_j) f_{m,21}(t_j, s_i) \delta_{ij} + \sum_{j=1}^M P_3(k_j) f_{m,31}(k_j, s_i) \delta_{kj} \right] +$$

$$\left[ \sum_{j=1}^M Q_2(t_j) f_{n,21}(t_j, s_i) \delta_{ij} + \sum_{j=1}^M Q_3(k_j) f_{n,31}(k_j, s_i) \delta_{kj} \right] = \sigma_1 \quad i = 1, 2, \dots, M$$

$$Q_1(s_i) + \left[ \sum_{j=1}^M P_2(t_j) f_{n,21}(t_j, s_i) \delta_{ij} + \sum_{j=1}^M P_3(k_j) f_{n,31}(k_j, s_i) \delta_{kj} \right] +$$

$$\left[ \sum_{j=1}^M Q_2(t_j) f_{m,21}(t_j, s_i) \delta_{ij} + \sum_{j=1}^M Q_3(k_j) f_{m,31}(k_j, s_i) \delta_{kj} \right] = \tau_1 \quad i = 1, 2, \dots, M \quad (10)$$

$$P_2(t_i) + \left[ \sum_{j=1}^M P_1(s_j) f_{m,12}(s_j, t_i) \delta_{sj} + \sum_{j=1}^M P_3(k_j) f_{m,32}(k_j, t_i) \delta_{kj} \right] +$$

$$\left[ \sum_{j=1}^M Q_1(s_j) f_{m,12}(s_j, t_i) \delta_{sj} + \sum_{j=1}^M Q_3(k_j) f_{m,32}(k_j, t_i) \delta_{kj} \right] = \sigma_2 \quad i = 1, 2, \dots, M$$

$$Q_2(t_i) + \left[ \sum_{j=1}^M P_1(s_j) f_{n,12}(s_j, t_i) \delta_{sj} + \sum_{j=1}^M P_3(k_j) f_{n,32}(k_j, t_i) \delta_{kj} \right] +$$

$$\left[ \sum_{j=1}^M Q_1(s_j) f_{n,12}(s_j, t_i) \delta_{sj} + \sum_{j=1}^M Q_3(k_j) f_{n,32}(k_j, t_i) \delta_{kj} \right] = \tau_2 \quad i = 1, 2, \dots, M \quad (11)$$

$$P_3(k_i) + \left[ \sum_{j=1}^M P_1(s_j) f_{m,13}(s_j, k_i) \delta_{sj} + \sum_{j=1}^M P_2(t_j) f_{m,23}(t_j, k_i) \delta_{tj} \right] +$$

$$\left[ \sum_{j=1}^M Q_1(s_j) f_{m,13}(s_j, k_i) \delta_{sj} + \sum_{j=1}^M Q_2(t_j) f_{m,23}(t_j, k_i) \delta_{tj} \right] = \sigma_3 \quad i = 1, 2, \dots, M$$

$$Q_3(k_i) + \left[ \sum_{j=1}^M P_1(s_j) f_{n,13}(s_j, k_i) \delta_{sj} + \sum_{j=1}^M P_2(t_j) f_{n,23}(t_j, k_i) \delta_{tj} \right] +$$

$$\left[ \sum_{j=1}^M Q_1(s_j) f_{n,13}(s_j, k_i) \delta_{sj} + \sum_{j=1}^M Q_2(t_j) f_{n,23}(t_j, k_i) \delta_{tj} \right] = \tau_3 \quad i = 1, 2, \dots, M \quad (12)$$

求解以上方程组即可求得 3 条裂纹相互影响下的  $P_1(s_i)$ ,  $Q_1(s_i)$ ,  $P_2(t_i)$ ,  $Q_2(t_i)$ ,  $P_3(k_i)$ ,  $Q_3(k_i)$ , 代入式(6), 计算得到不同裂纹倾角下裂纹尖端各点的应力强度因子值。将考虑裂纹间相互影响计算得到的结果与式(9)进行对比, 得到无量纲应力强度因子值, 见图 6。

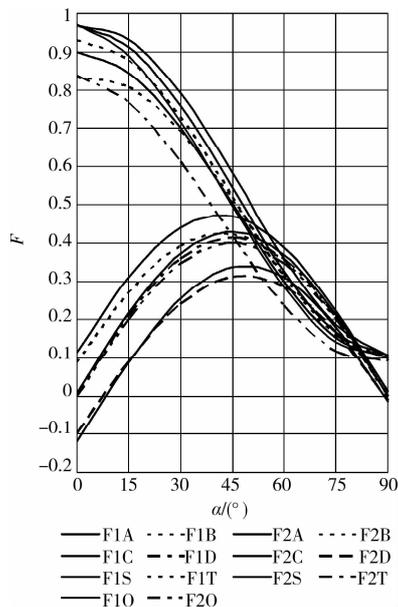


图 6 3 条裂纹 F1A, F1B..., F2T 值

图中  $F$  各值如下式:

$$K_{I,A} = F_{I,A}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,A} = F_{II,A}(\alpha) \sigma \sqrt{\pi a},$$

$$K_{I,B} = F_{I,B}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,B} = F_{II,B}(\alpha) \sigma \sqrt{\pi a},$$

$$K_{I,C} = F_{I,C}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,C} = F_{II,C}(\alpha) \sigma \sqrt{\pi a}, \quad (13)$$

$$K_{I,D} = F_{I,D}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,D} = F_{II,D}(\alpha) \sigma \sqrt{\pi a},$$

$$K_{I,S} = F_{I,S}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,S} = F_{II,S}(\alpha) \sigma \sqrt{\pi a},$$

$$K_{I,T} = F_{I,T}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,T} = F_{II,T}(\alpha) \sigma \sqrt{\pi a}$$

$$K_{I,O} = F_{I,O}(\alpha) \sigma \sqrt{\pi a}, \quad K_{II,O} = F_{II,O}(\alpha) \sigma \sqrt{\pi a}$$

由图 4、图 6 可知, 多裂纹的相互作用对裂纹尖端处的应力场影响较大, 根据裂纹的相对几何位置和裂纹倾角变化, 进行不同叠加后的应力场可产生 2 种效应, 应力强度影响加强区与应力强度影响减弱区, 当相邻裂纹尖端处于加强区时, 该裂纹的应力强度因子将会变大, 反之则变小。

### 3 渗透压下多裂纹相互作用分析

计算模型见图 3, 按裂纹渗透水压  $p=0, 2, 4, 6, 8, 10$  MPa; 裂纹间垂直距离  $2d=2$  mm, 水平距离  $2b=0, 2, 4, 6, 8, 10$  mm 共 36 种情况进行计算, 得到了各种情况下裂纹尖端的应力强度因子, 如图 7—8 所示。

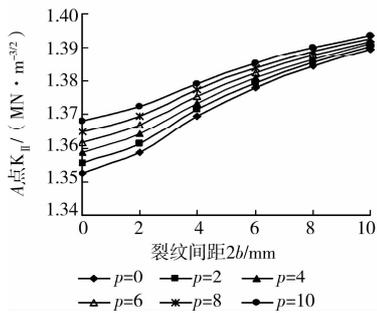
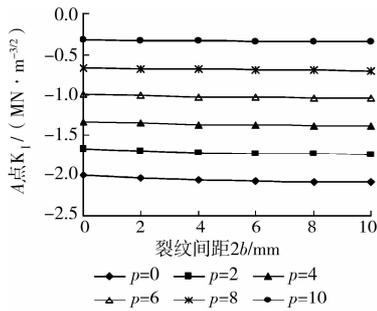


图 7 裂纹 1 尖端 A 点  $K_I$ 、 $K_{II}$  值

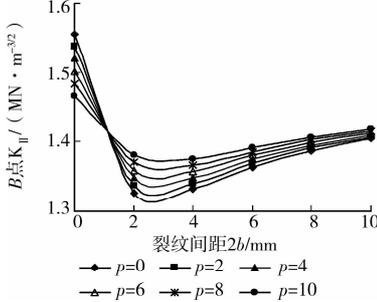
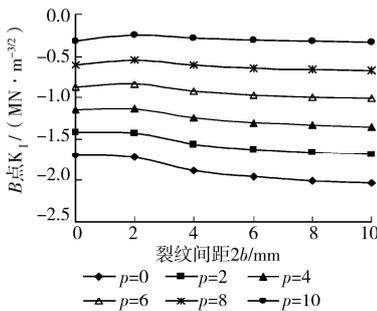


图 8 裂纹 1 尖端 B 点  $K_I$ 、 $K_{II}$  值

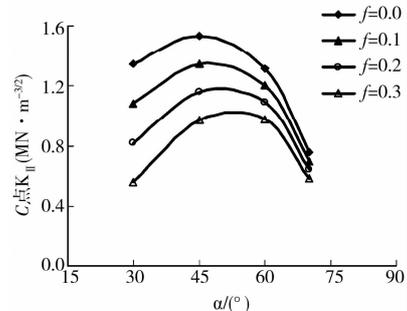
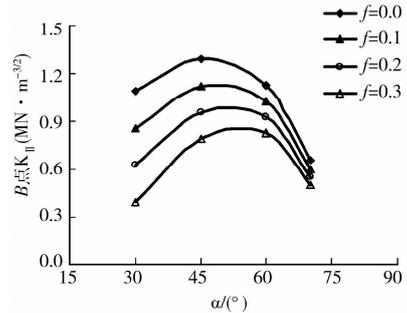
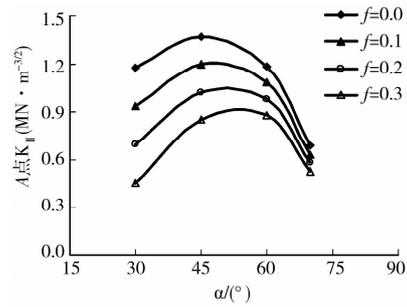


图 9 摩擦系数对  $K_{II}$  的影响

从上图可知,各条裂纹尖端  $K_I$  随着渗透压的增大而增加,即使  $-K_I$  逐渐减小,这说明裂隙水压力抵消了一部分正应力。裂纹外端部 A 点的  $K_{II}$  有相同的变化趋势,而内端部 B 点  $K_{II}$  的变化还受到相邻裂纹尖端应力场的影响,随着裂纹水平间距的增大出现先减小后增大的规律,但最终都趋于一个定值,该值为考虑单一裂纹计算所得到的  $K_{II}$ 。

对于闭合裂纹,其端部 I 型应力强度因子恒为零,由裂纹上下表面间摩擦力的存在影响了裂纹面的相对滑动,从而影响应力强度因子  $K_{II}$ ,见图 9。从图中可知,2 条裂纹尖端的应力强度因子变化趋势相同,都随摩擦系数的增大而减小。

## 4 结 论

1)岩体中的裂隙大都成组出现,每一个裂纹都被其他裂隙所包围,这些都会影响裂纹尖端附近的应力状态。多裂纹的相互作用对裂纹尖端处的应力场影响较大,根据裂纹的相对几何位置和裂纹倾角变化,进行不同叠加后的应力场可产生 2 种效应,应力强度影响加强区与应力强度影响减弱区,当相邻裂纹尖端处于加强区时,该裂纹的应力强度因子将会变大,反之则变小。

2)渗透水压的存在,大大的改变了裂隙尖端的应力强度因子,各条裂纹尖端  $K_I$  随着渗透压的增大而增加(使  $-K_I$  逐渐减小),这说明裂隙水压力抵消了一部分正应力。

3)不同间距的平行裂纹相互影响所产生的应力强度因子的变化趋势是完全一致的,且随着裂纹间距离的加大,计算所得出的应力强度因子值都趋于一个定值,该值为没有裂纹影响时的应力强度因子值。考虑裂纹间的相互作用与裂纹间的相对距离有关,当裂纹间距>裂纹长度时,可以忽略裂纹相互作用对裂纹尖端应力场的影响。

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#### 附录 A 裂纹相互作用系数和相互作用因子

如图 A1 所示,无限大平板内有一条长为  $2a$  的裂纹  $AB$ ,坐标原点取于裂纹中点,  $x$  轴与裂纹重合,在裂纹附近考虑一虚拟裂纹  $CD$ ,与  $x$  轴的夹角为  $\alpha$ ,分析虚拟裂纹  $CD$  上任意一点的应力情况。

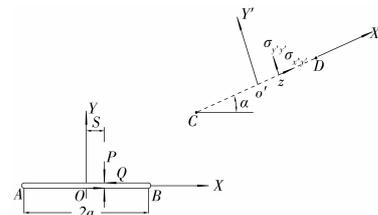


图 A1 虚拟裂纹计算

式(3)、式(4)中相互作用系数和相互作用因子可以由弹性力学基本解求得如下<sup>[A1, A2]</sup>:

$$\sigma_{y'y'} - i\sigma_{x'y'} = f_m - if_u = -\frac{\sqrt{a^2 - s^2}}{2\pi} \cdot [G(z) + \overline{G(z)} + e^{-2ia}(z - \bar{z}) \overline{G'(z)}]$$

$$\sigma_{y'y'} - i\sigma_{x'y'} = f_m - if_u = \frac{\sqrt{a^2 - s^2}}{2\pi i} [G(z)(1 - 2e^{-2ia}) - G(z) + e^{-2ia}(z - \bar{z}) \overline{G'(z)}] \quad (14)$$

$$X(z) = \sqrt{z^2 - a^2}$$

$$G(z) = \frac{1}{X(z)(z - s)}$$

$$G'(z) = \frac{a^2 + sz - 2z^2}{(z - s)^2 (X(z))^3} \quad (15)$$

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