重庆建筑工程学院学报
第13卷第3期 JOURNAL OF CHONGQING INSTITUTE OF Vol.13 No.3
1991年9月 ARCHITECTURE AND ENGINEERING Sept.1991

高阶剪切变形理论下角铺设层合板的大挠度问题

刘 东 欧茂材 刘龙泉

（建筑工程系）

摘 要 本文根据Reddy的三阶剪切变形理论，导出了该理论下对称角铺设层合板非线性弯曲问题的控制微分方程组，并在四边固支边界条件下，用摄动法对上述微分方程组进行了求解。针对不同的层合板参数进行了大量的数值计算，并与经典弯曲理论下的结果进行了比较，得到一些有意义的结果。

关键词 剪切变形，非线性弯曲，层合板

用板的经典弯曲理论(CPT)来分析，计算板的弯曲问题时，所得结果与实验数据不能很好吻合。其主要原因是经典弯曲理论忽略了横向剪切变形的影响，从而低估了板的挠度。由于复合材料层合板的拉伸弹性模量比剪切弹性模量大很多，因此更应该考虑剪切变形的影响。已有不少学者提出了多种剪切变形理论。1983年J.N.Reddy提出了“三阶剪切变形理论”[1][2]。该理论从位移场假设出发，且自动满足板上、下表面剪应力为零的条件，因而不必引入“剪切校正系数”，比以往的“一阶剪切变形理论”更合理。本文用摄动法求解了该理论下四边固定对称角铺设层合矩形板的非线性弯曲问题。计算结果表明：横向剪切变形的影响程度与层合板的宽厚比有很大关系，同时也与铺设角和材料特性有关。

1 三阶剪切变形理论和控制微分方程组

该理论的位移场假设为[2]：

$$u_1 = u + Z\left[\psi_x - \frac{4}{3}\left(\frac{Z}{h}\right)^2(\psi_x + w_{,x})\right]$$

$$u_2 = v + Z\left[\psi_y - \frac{4}{3}\left(\frac{Z}{h}\right)^2(\psi_y + w_{,y})\right]$$

$$u_3 = w \tag{1}$$

本文1990年10月12日收到。

式中u，v，w是中面位移。ψ_x，ψ_y分别为中面法线关于y轴和x轴的转角，h为板厚。

应变——位移关系为：

$$\varepsilon_1 = u_{,x} + \frac{1}{2}w_{,x}^2 + Z\left[\psi_{x,x} - \frac{4}{3h^2}Z^2(\psi_{x,x} + w_{,xx})\right]$$

$$\varepsilon_2 = v_{,y} + \frac{1}{2}w_{,y}^2 + Z\left[\psi_{y,y} - \frac{4}{3h^2}Z^2(\psi_{y,y} + w_{,yy})\right.$$

$$\varepsilon_3 = 0$$

$$\varepsilon_4 = (\psi_y + w_{,y}) - \frac{4}{h^2}Z^2(\psi_y + w_{,x})$$

$$\varepsilon_5 = (\psi_x + w_{,x}) - \frac{4}{h^2}Z^2(\psi_x + w_{,x})$$

$$\varepsilon_6 = u_{,y} + v_{,x} + w_{,x}w_{,y} + Z\left[\psi_{x,y} + \psi_{y,x} - \frac{4}{3h^2}Z^2(\psi_{x,y} + \psi_{y,x} + 2w_{,xy})\right] \qquad (2)$$

板坐标系下的应力——应变关系为

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{Bmatrix} = \begin{vmatrix} Q_{11} & Q_{12} & Q_{16}\\ Q_{12} & Q_{22} & Q_{26}\\ Q_{16} & Q_{26} & Q_{66}\end{vmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_4\\ \sigma_5\end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45}\\ Q_{45} & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_4\\ \varepsilon_5\end{Bmatrix} \qquad (3)$$

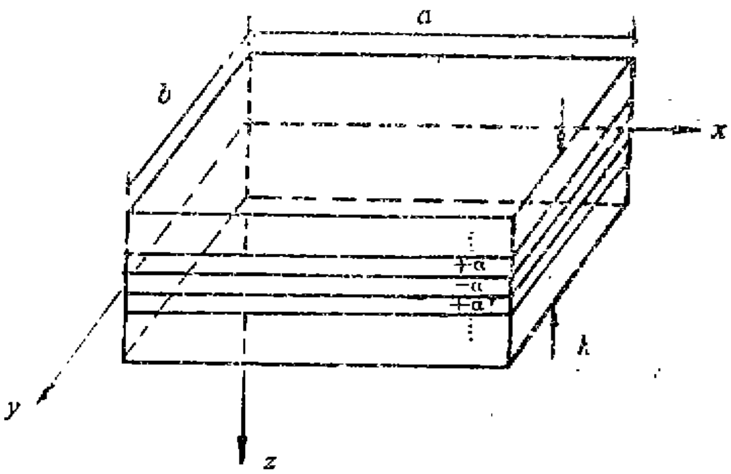

图 1

图 1 给出了层合板的几何尺寸和铺层形式。

由Hamiltan原理即可导出与位移场（1）和本构关系（3）相适应的平衡微分方程组。引入适当的无量纲量和对称角铺设层合板的刚度系数，即可得如下对称角铺设层合板的无量纲控制微分方程组：

$$L_1U + L_2V + \lambda_1^{-1}W_{,\xi}L_1W + W_{,\eta}L_2W = 0$$

$$L_2U + L_3V + \lambda_1^{-1}W_{,\xi}L_2W + W_{,\eta}L_3W = 0$$

$$L_4W+\lambda_1L_5\phi_x+\lambda_1L_6\phi_y+D_{11}Q+h^2\Big[(\lambda_1U_{,\xi}+\frac{1}{2}W_{,\xi}^2)L_{13}W+\lambda_1(\lambda_1U_{,\eta}+V_{,\xi}+W_{,\xi}W_{,\eta})L_{14}W+\lambda_1^2(V_{,\eta}+\frac{1}{2}W_{,\eta}^2)L_{15}W+W_{,\xi}^2L_{16}W+2\lambda_1W_{,\xi}W_{,\eta}L_{17}W+\lambda_1^2W_{,\eta}^2L_{18}W+\lambda_1U_{,\xi\xi}L_{19}W+\lambda_1^2U_{,\xi\eta}L_{20}W+\lambda_1^3U_{,\eta\eta}L_{21}W+\lambda_1V_{,\xi\xi}L_{22}W+\lambda_1^2V_{,\xi\eta}L_{23}W+\lambda_1^3V_{,\eta\eta}L_{24}W\Big]=0$$

$$L_7W+\lambda_1L_8\phi_x+\lambda_1L_9\phi_y=0$$

$$L_{10}W+\lambda_1L_{11}\phi_x+\lambda_1L_{12}\phi_y=0 \qquad (4)$$

式中$L_i(i=1,2,\cdots24)$是微分算子，见附录。

（4）式中的各无量纲量定义如下：

$$\xi=x/a,\ \eta=y/b,\ U=bu/h^2,\ V=bv/h^2$$

$$W=w/h,\ \Phi_x=\Psi_xb/h,\ \Phi_y=\Psi_yb/h,\ \lambda_1=a/b$$

$$\lambda_2=a/h^2,\ Q=a^4q_0/D_{11}h \qquad (5)$$

无量纲边界条件为：

$$\xi=0,\ 1;\quad U=V=W=W_{,\xi}=\phi_x=\phi_y=0$$

$$\eta=0,\ 1;\quad U=V=W=W_{,\eta}=\phi_x=\phi_y=0 \qquad (6)$$

板承受法向均布荷载作用，分布集度为q_0

2 摄动——伽辽金法求解

取板的无量纲中心挠度$W(0.5,\ 0.5)$为摄动参数，记为W_0，将无量纲量Q，W，Φ_x，Φ_y，U，V展成W_0的级数形式：

$$Q=\sum_{m=1,3\cdots}q_mW_0^m$$

$$W=\sum_{m=1,3\cdots}w_m(\xi,\eta)W_0^m$$

$$\phi_x=\sum_{m=1,3\cdots}x_m(\xi,\eta)W_0^m$$

$$\phi_y=\sum_{m=1,3\cdots}y_m(\xi,\eta)W_0^m$$

$$U=\sum_{m=2,4,\cdots}u_m(\xi,\eta)W_0^m$$

$$V=\sum_{m=2,4,\cdots}v_m(\xi,\eta)W_0^m \qquad (7)$$

虑边界条件为：

$$w_1(0.5,\ 0.5)=1$$

$$w_m(0.5,\ 0.5)=0\quad(m\neq1) \qquad (8)$$

将摄动级数(7)代入控制微分方程组(4)和边界条件(6)中，并令W_0同次幂的系数相等，即可得各级摄动微分方程组和相应的边界条件：

$$L_7w_m+\lambda_1L_8x_m+\lambda_1L_9y_m=0$$

$$L_{10}w_m+\lambda_1L_{11}x_m+\lambda_1L_{12}y_m=0$$

$$L_4 w_m+\lambda_1 L_5 x_m+\lambda_1 L_6 y_m+D_{11} q_m+h^2\sum_{i=1,3\cdots}^{m-2}\Big[\lambda_1 u_{m-i,\xi} L_{13} w_i+\lambda_1(\lambda_1 u_{m-i,\eta}+v_{m-i,\xi}) L_{14} w_i+\lambda_1 v_{m-i,\eta} L_{15} w_i+\lambda_1 u_{m-i,\xi\xi} L_{19} w_i+\lambda_1^2 u_{m-i,\xi\eta} L_{20} w_i+\lambda_1^3 u_{m-i,\eta\eta} L_{21} w_i+\lambda_1 v_{m-i,\xi\xi} L_{22} w_i+\lambda_1^2 v_{m-i,\xi\eta} L_{23} w_i+\lambda_1^3 v_{m-i,\eta\eta} L_{24} w_i\Big]+h^2\sum_{i=1,3\cdots}^{m-2}\sum_{j=1,3\cdots}^{m-i-1}\Big[w_{i,\xi} w_{j,\xi}(\frac{1}{2} L_{13} w_{m-i-j}+L_{16} w_{m-i-j})+\lambda_1 w_{i,\xi} w_{j,\eta}(L_{14} w_{m-i-j}+2L_{17} w_{m-i-j})+\lambda_1^2 w_{i,\eta} w_{j,\eta}(\frac{1}{2} L_{15} w_{m-i-j}+L_{18} w_{m-i-j})\Big]=0 \tag{9}$$

$$\begin{aligned}&\xi=0,1\quad w_m=w_{m,\xi}=x_m=y_m=0\\&\eta=0,1\quad w_m=w_{m,\eta}=x_m=y_m=0\\&\qquad (m=1,3,\cdots)\end{aligned}\tag{10}$$

和

$$\begin{aligned}&L_1 u_m+L_2 v_m+\sum_{i=1,3\cdots}^{m-1}\Big(\frac{1}{\lambda_1} w_{i,\xi} L_1 w_{m-i}+w_{i,\eta} L_2 w_{m-i}\Big)=0\\&L_2 u_m+L_3 v_m+\sum_{i=1,3\cdots}^{m-1}\Big(\frac{1}{\lambda_1} w_{i,\xi} L_2 w_{m-i}+w_{i,\eta} L_3 w_{m-i}\Big)=0\end{aligned}\tag{11}$$

$$\xi=0,1\ \ \text{and}\ \ \eta=0,1\quad u_m=v_m=0\qquad(m=2,4,\cdots)\tag{12}$$

各级摄动解均取为如下双级数形式：

$$\begin{aligned}&w_m(\xi,\eta)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} w_{pq}^{(m)}\, wx_p(\xi)\, wy_q(\eta)\\&x_m(\xi,\eta)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} x_{pq}^{(m)}\, S_p(\xi)\, wy_q(\eta)\\&y_m(\xi,\eta)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} y_{pq}^{(m)}\, wx_p(\xi)\, T_q(\eta)\\&\qquad(m=1,3\cdots)\end{aligned}\tag{13}$$

$$\begin{aligned}&u_m(\xi,\eta)=\sum_{p=1}\sum_{q=1}^{\infty} u_{pq}^{(m)}\, S_p(\xi)\, wy_q(\eta)\\&v_m(\xi,\eta)=\sum_{p=1}\sum_{q=1} v_{pq}^{(m)}\, wx_p(\xi)\, T_q(\eta)\\&\qquad(m=2,4,\cdots)\end{aligned}\tag{14}$$

式中$wx_p(\xi)$与$wy_q(\eta)$和$S_p(\xi)$与$T_q(\eta)$具有相同的形式：

$$\begin{aligned}&wx_p(\xi)=ch\lambda_p\xi-\cos\lambda_p\xi-\frac{ch\lambda_p-\cos\lambda_p}{sh\lambda_p-\sin\lambda_p}(sh\lambda_p\xi-\sin\lambda_p\xi)\\&S_p(\xi)=\sin 2p\pi\xi\end{aligned}\tag{15}$$

当λ_p按表1取值时，所有的边界条件均能满足。

表 1　λ_p的值

P	1	2	3	4	>4
λ_p	4.7300	7.8532	10.9956	14.1372	$\frac{(2P+1)\pi}{2}$

将（13），（14）代入（9）和（11）中，得到残值R_i（$i=1, 2\cdots5$）。将残值R_i代入伽辽金方程可得消除残值的方程组：

$$\int_0^1\int_0^1 R_i f_i d\xi d\eta = 0 \quad (i=1, 2, 3, 4, 5) \tag{16}$$

其中

$$(f_1, f_2, f_3, f_4, f_5) = s_p(\xi)wy_q(\eta),\ wx_p(\xi)T_q(\eta),\ wx_p(\xi)wy_q(\eta),\ s_p(\xi)wy_q(\eta),\ wx_p(\xi)T_q(\eta))$$

(16)式给出了确定各次摄动解中待定系数$\left\{x_{pq}^{(m)}\right\}$，$\left\{y_{pq}^{(m)}\right\}$，$\left\{w_{pq}^{(m)}\right\}$和$\left\{u_{pq}^{(m)}\right\}$，$\left\{v_{pq}^{(m)}\right\}$的两组线性代表方程组。其矩阵形式分别为：

$$AX^{(m)} = C^{(m)} \quad (m=1, 3, \cdots) \tag{17}$$

$$BY^{(m)} = D^{(m)} \quad (m=2, 4, \cdots) \tag{18}$$

其中A，B是系数矩阵。$X^{(m)} = \left[\left\{x_{pq}^{(m)}\right\}, \left\{y_{pq}^{(m)}\right\}, \left\{w_{pq}^{(m)}\right\}\right]^T$，$Y^{(m)} = \left[\left\{u_{pq}^{(m)}\right\}, \left\{v_{pq}^{(m)}\right\}\right]^T$是未知量列阵。$C^{(m)}$，$D^{(m)}$是常数项列阵，其值随$m$而异。求解时（12）和（13）中的各级数均取49项。

求解过程为，在（17）中取$m=1$，并考虑到（8）中第一式，可求出$X^{(1)}$和q_1，得到一级摄动解。又在(18)中取$m=2$，可求出$Y^{(2)}$，得到二级摄动解。再在（17）中取$m=3$，并考虑到（8）中第二式，可求出$Z^{(3)}$和q_3，从而获得三级摄动解。更高级的摄动解可类似得到。本文只计算到问题的三级摄动解。

由应变——位移关系（2），应力——应变关系（3）可以求出层合板的薄膜应力和弯曲应力：无量纲薄膜应力

$$\bar{\sigma}_1^m = \lambda_1\left(U_{,\xi} + \frac{Q_{16}}{Q_{11}}V_{,\xi} + \frac{Q_{16}}{Q_{11}}W_{,\xi}W_{,\eta}\right) + \lambda_1^2\left(\frac{Q_{16}}{Q_{11}}U_{,\eta} + \frac{Q_{12}}{Q_{11}}V_{,\eta} + \frac{1}{2}\frac{Q_{12}}{Q_{11}}W_{,\eta}^2\right) + \frac{1}{2}W_{,\xi}^2$$

无量纲 最大弯曲应力

$$\bar{\sigma}_1^b = \frac{1}{6}\left[2\lambda_1\left(\phi_{x,\xi} + \frac{Q_{16}}{Q_{11}}\phi_{y,\xi} - \frac{Q_{16}}{Q_{11}}W_{,\xi\eta}\right) + \lambda_1^2\left(2\frac{Q_{16}}{Q_{11}}\phi_{x,\eta} + 2\frac{Q_{12}}{Q_{11}}\phi_{y,\eta} - \frac{Q_{12}}{Q_{11}}W_{,\eta\eta}\right) - W_{,\xi\xi}\right]$$

其中

$$(\bar{\sigma}_1^m, \bar{\sigma}_1^b) = \frac{a^2}{Q_{11}h^2}(\sigma_1^m, \sigma_1^b) \tag{19}$$

其余的应力可类似求得。

3 计算结果与分析

本文对三阶剪切变形理论下（HSDT）四边简支对称角铺设层合板的大挠度问题进行了计算。材料常数取为：

M_a.Ⅰ：$E_1/E_2=40$，$G_{12}/E_2=G_{13}/E_2=0.6$，$G_{23}/E_2=0.2$，$\nu_{12}=\nu_{13}=0.25$

M_a.Ⅱ：$E_1/E_2=25$，$G_{12}/E_2=G_{13}/E_2=0.5$，$G_{23}/E_2=0.2$，$\nu_{12}=\nu_{13}=0.25$

作者还根据文献〔5〕的方法编制了经典弯曲理论（CPT）下的计算程序，其计算结果与文献〔5〕，〔6〕相当吻合。

在下面给出的算例中，我们取$a_0=a/2$。

图 2 给出的是三阶剪切变形理论下的荷载——挠度曲线，并与经典弯曲理论下的结果进行了比较。从图中可以看到，经典弯曲理论低估了层合板的挠度。

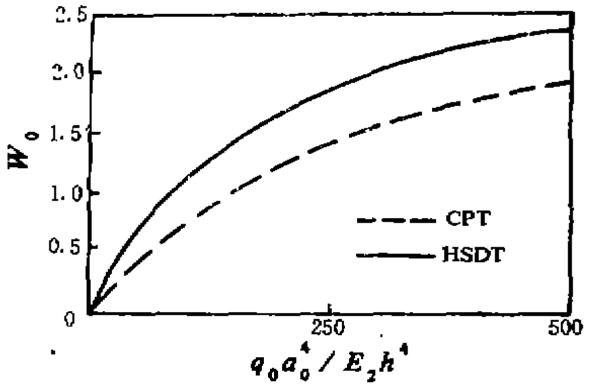

图 2　三阶剪切变形理论下的荷载—挠度曲线

$\left(\frac{a_0}{h}=5, \frac{a}{b}=1, NL=3, \theta=15°, M_a.I\right)$

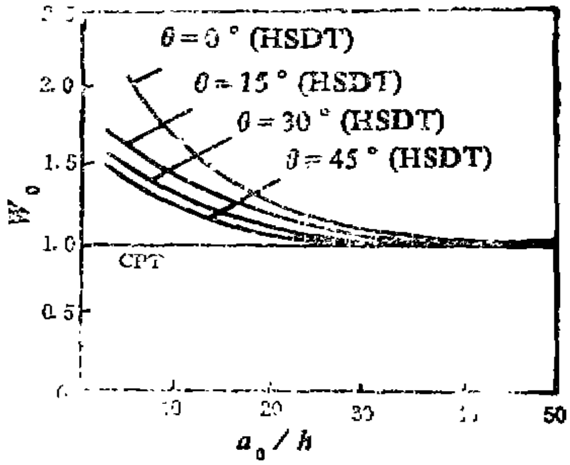

图 3　不同铺设角时层合板宽厚比对挠度的影响

$\left(NL=3, \frac{a}{b}=1, M_a.I\right)$

图 3 给出的是不同铺设角下挠度随宽厚比变化的曲线。可以看到，在宽厚比较小时，由三阶剪切变形理论得到的挠度与经典弯曲理论下的挠度有较大的差别。且剪切变形的影响程

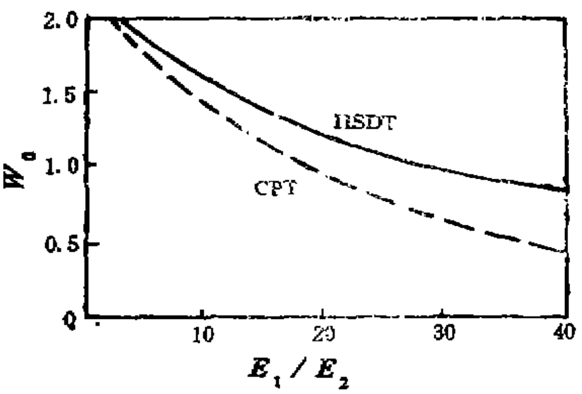

图 4　材料各向异性对层合板挠度的影响

$\left(\frac{a_0}{h}=5, \frac{a}{b}=1, NL=3, \theta=15°\right)$

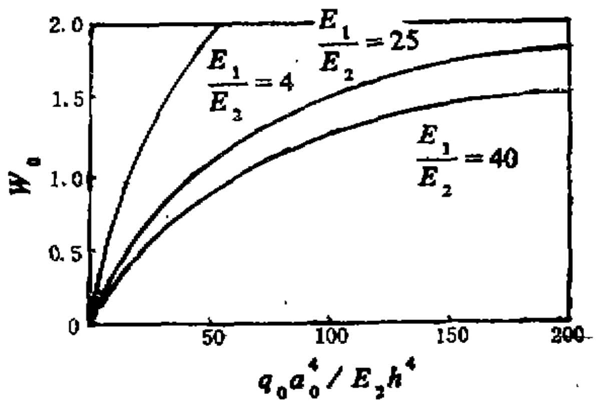

图 5　不同材料的荷载—挠度曲线

$\left(\frac{a_0}{h}=5, \frac{a}{b}=1, NL=3, \theta=15°\right)$

度与铺设角有关；随着铺设角的减小，剪切变形的影响程度越来越大。

图 4 给出的是三层、15°角铺设层合板的弹性模量比——挠度曲线。结果表明，剪切变形的影响程度与材料的各向异性程度有关，它随材料弹性模量比的减小而减小。即使在弹性模量比较小的情况下，仍应考虑剪切变形的影响。图 5 给出的是不同弹性模量比下的荷载一挠度曲线。

图 6 给出的是不同铺设角下的荷载—挠度曲线。图7给出的是45°角铺设层合板在不同铺层数下的荷载—挠度曲线。

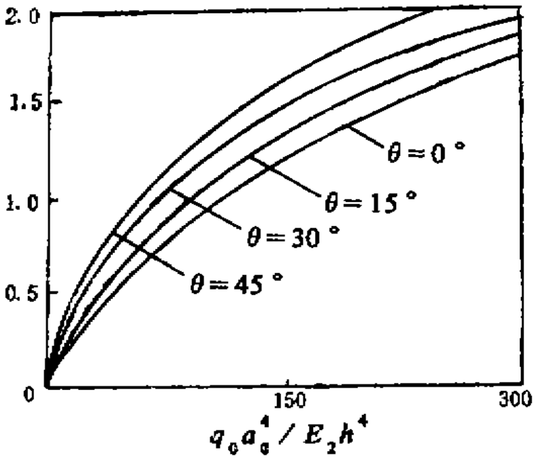

图 6 三阶剪切变形理论下不同铺设角层板的荷合载—挠度曲线

$\left(\frac{a_0}{h}=5,\ \frac{a}{b}=1, \theta=45°, M_a.I\right)$

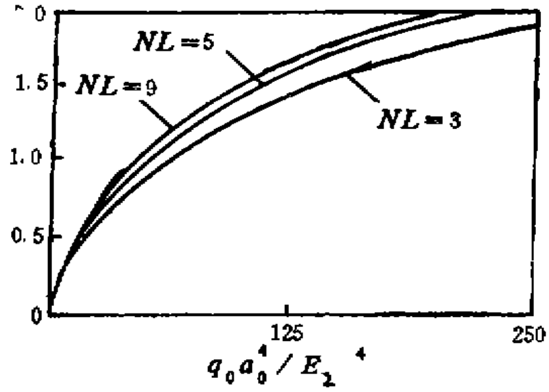

图 7 三阶剪切变形理论下不同铺设层数层合板的荷载—挠度曲线

$\left(\frac{a_0}{h}=5,\ \frac{a}{b}=1, \theta=45°, M_a{\cdot}I\right)$

表2列出的是30°角铺设层合板在不同边长比时的荷载值。图 8 给出的是铺设角为0°时的荷载—挠度曲线。图形表明，随着宽厚比的增大非线性项的影响增大。

表 2 不同长宽比时的荷载值

W_0	$q_0a_0^4/E_2h^4$			
	$\frac{a}{b}=1.0$	$\frac{a}{b}=1.5$	$\frac{a}{b}=2.0$	$\frac{a}{b}=3.0$
0.6	20.89	17.01	16.25	15.56
1.0	56.01	46.56	45.10	43.25
1.6	172.25	145.56	142.25	136.69
2.0	310.69	264.21	258.75	248.68

$(\frac{a_0}{h}=5,\ NL=3,\ \theta=30°,\ M_a.I)$

图9，图10给出的是30°角铺设层合板的薄膜应力和弯曲应力随挠度变化的曲线。计算结

果表明，薄膜应力和弯曲应力的最大值均发生在固支边界上，横向剪切应力与薄膜应力和弯曲应力相比很小。

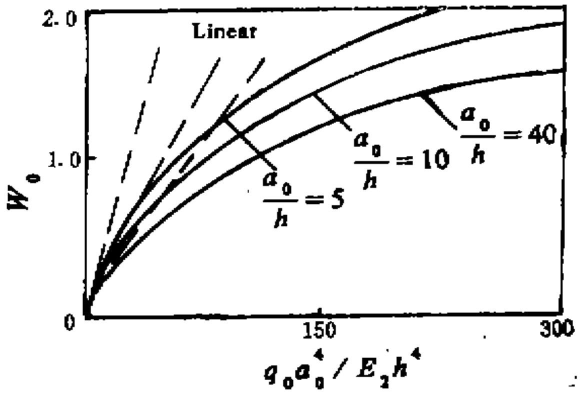

图 8　三阶剪切变形理论下不同宽厚比层合板的荷载—挠度曲线

$\left(\frac{a}{b}=1, NL=3, \theta=0°, M_a.I\right)$

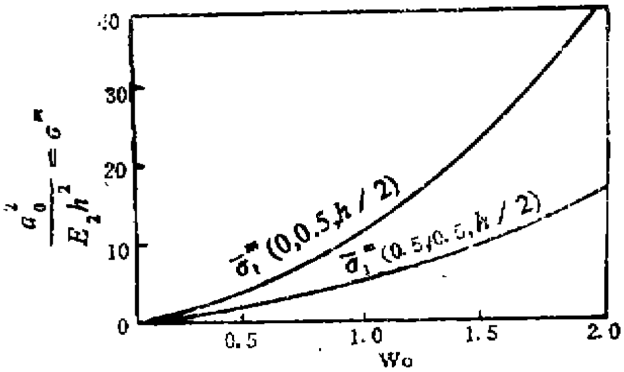

图 9　三阶剪切变形理论下薄膜应力变化曲线

$\left(\frac{a_0}{h}=5, \frac{a}{b}=1, NL=3, \theta=30°, M_a.I\right)$

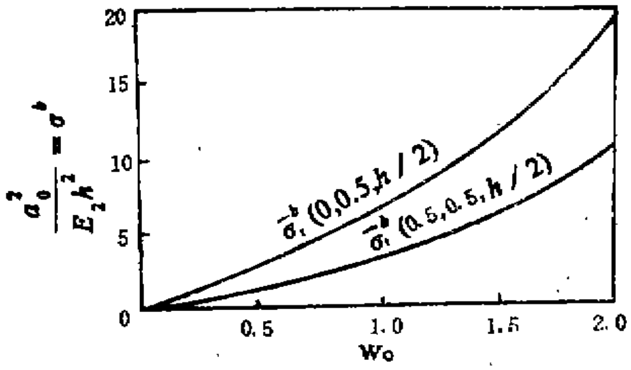

图10　三阶剪切变形理论下弯曲应力变化曲线

$\left(\frac{a_0}{h}=5, \frac{a}{b}=1, NL=3, \theta=30°, M_a.I\right)$

本文计算是在SIEMENS 7570C机上完成的。

附　录

（4）式中微分算子L_i定义如下：

$L_1(\ )==A_{11}(\ )_{,\xi\xi}+2A_{16}\lambda_1(\ )_{,\xi\eta}+A_{66}\lambda_1^2(\ )_{,\eta\eta}$

$L_2(\ )=A_{16}(\ )_{,\xi\xi}+(A_{12}+A_{66})\lambda_1(\ )_{,\xi\eta}+A_{26}\lambda_1(\ )_{,\eta\eta}$

$L_3(\ )=A_{66}(\ )_{,\xi\xi}+2A_{26}\lambda_1(\ )_{,\xi\eta}+A_{22}\lambda_1^2(\ )_{,\eta\eta}$

$$L_4(\ ) = \bar{A}_{55}a^2(\ )_{,\xi\xi} + 2\bar{A}_{45}a^2\lambda_1(\ )_{,\xi\eta} + \bar{A}_{44}a^2\lambda_1^2(\ )_{,\eta\eta} - \frac{16}{9h^4}\Big[H_{11}(\ )_{,\xi\xi\xi\xi} + 4H_{16}\lambda_1(\ )_{,\xi\xi\xi\eta} + 2(H_{12}+2H_{66})\lambda_1^2(\ )_{,\xi\xi\eta\eta} + 4H_{26}\lambda_1^3(\ )_{,\xi\eta\eta\eta} + H_{22}\lambda_1^4(\ )_{,\eta\eta\eta\eta}\Big]$$

$$L_5(\ ) = \frac{4}{3h^2}\Big[\bar{F}_{11}(\ )_{,\xi\xi\xi} + 3\bar{F}_{16}\lambda_1(\ )_{,\xi\xi\eta} + (2\bar{F}_{66}+\bar{F}_{12})\lambda_1^2(\ )_{,\xi\eta\eta} + \bar{F}_{26}\lambda_1^3(\ )_{,\eta\eta\eta}\Big] + \bar{A}_{55}a^2(\ )_{,\xi} + \bar{A}_{45}a^2\lambda_1(\ )_{,\eta}$$

$$L_6(\ ) = \frac{4}{3h^2}\Big[\bar{F}_{16}(\ )_{,\xi\xi\xi} + (2\bar{F}_{66}+\bar{F}_{12})\lambda_1(\ )_{,\xi\xi\eta} + 3\bar{F}_{26}\lambda_1^2(\ )_{,\xi\eta\eta} + \bar{F}_{22}\lambda_1^3(\ )_{,\eta\eta\eta}\Big] + \bar{A}_{45}a^2(\ )_{,\xi} + \bar{A}_{44}a^2\lambda_1(\ )_{,\eta}$$

$$L_7(\ ) = \frac{4}{3h^2}\Big[-\bar{F}_{11}(\ )_{,\xi\xi\xi} - 3\bar{F}\lambda_1(\ )_{,\xi\xi\eta} - (2\bar{F}_{66}+\bar{F}_{12})\lambda_1^2(\ )_{,\xi\eta\eta} - \bar{F}_{26}\lambda_1^3(\ )_{,\eta\eta\eta}\Big] - \bar{A}_{55}a^2(\ )_{,\xi} - \bar{A}_{45}a^2\lambda_1(\ )_{,\eta}$$

$$L_8(\ ) = \bar{D}_{11}(\ )_{,\xi\xi} + 2\bar{D}_{16}\lambda_1(\ )_{,\xi\eta} + \bar{D}_{66}\lambda_1^2(\ )_{,\eta\eta} - \bar{A}_{55}a^2(\ )$$

$$L_9(\ ) = \bar{D}_{16}(\ )_{,\xi\xi} + (\bar{D}_{12}+\bar{D}_{66})\lambda_1(\ )_{,\xi\eta} + \bar{D}_{26}\lambda_1^2(\ )_{,\eta\eta} - \bar{A}_{45}a^2(\ )$$

$$L_{10}(\ ) = \frac{4}{3h^2}\Big[-H_{16}(\ )_{,\xi\xi\xi} - (\bar{F}_{12}+\bar{F}_{66})\lambda_1(\ )_{,\xi\xi\eta} - 3\bar{F}_{26}\lambda_1^2(\ )_{,\xi\eta\eta} - \bar{F}_{22}\lambda_1^3(\ )_{,\eta\eta\eta}\Big] - \bar{A}_{45}a_1(\ )_{,\xi} - \bar{A}_{44}a^2\lambda_1(\ )_{,\eta}$$

$$L_{11}(\ ) = \bar{D}_{16}(\ )_{,\xi\xi} + (\bar{D}_{66}+\bar{D}_{12})\lambda_1(\ )_{,\xi\eta} + \bar{D}_{26}\lambda_1^2(\ )_{,\eta\eta} - \bar{A}_{45}a^2(\ )$$

$$L_{12}(\ ) = \bar{D}_{66}(\ )_{,\xi\xi} + 2\bar{D}_{26}\lambda_1(\ )_{,\xi\eta} + \bar{D}_{22}\lambda_1^2(\ )_{,\eta\eta} - \bar{A}_{44}a^2(\ )$$

$$L_{13}(\ ) = A_{11}(\ )_{,\xi\xi} + 2\lambda_1 A_{16}(\ )_{,\xi\eta} + \lambda_1^2 A_{12}(\ )_{,\eta\eta}$$

$$L_{14}(\ ) = A_{16}(\ )_{,\xi\xi} + 2\lambda_1 A_{66}(\ )_{,\xi\eta} + \lambda_1^2 A_{26}(\ )_{,\eta\eta}$$

$$L_{15}(\ ) = A_{12}(\ )_{,\xi\xi} + 2\lambda_1 A_{26}(\ )_{,\xi\eta} + \lambda_1^2 A_{22}(\ )_{,\eta\eta}$$

$$L_{16}(\ ) = A_{11}(\ )_{,\xi\xi} + 2\lambda_1 A_{16}(\ )_{,\xi\eta} + A_{66}\lambda_1^2(\ )_{,\eta\eta}$$

$$L_{17}(\ ) = A_{16}(\ )_{,\xi\xi} + (A_{12}+A_{66})\lambda_1(\ )_{,\xi\eta} + A_{26}\lambda_1^2(\ )_{,\eta\eta}$$

$$L_{18}(\ ) = A_{66}(\ )_{,\xi\xi} + 2A_{26}\lambda_1(\ )_{,\xi\eta} + \lambda_1^2 A_{22}(\ )_{,\eta\eta}$$

$$L_{19}(\ ) = A_{11}(\ )_{,\xi} + \lambda_1 A_{16}(\ )_{,\eta}$$

$$L_{20}(\ ) = 2A_{16}(\ )_{,\xi} + \lambda_1 A_{66}(\ )_{,\eta} + \lambda_1 A_{12}(\ )_{,\eta}$$

$$L_{21}(\ ) = A_{66}(\ )_{,\xi} + \lambda_1 A_{26}(\ )_{,\eta}$$

$L_{22}(\ )=A_{16}(\ )_{,\xi}+\lambda_1 A_{66}(\ )_{,\eta}$

$L_{23}(\ )=A_{12}(\ )_{,\xi}+2\lambda_1 A_{26}(\ )_{,\xi}+A_{66}(\ )_{,\xi}$

$L_{24}(\ )=A_{26}(\ )_{,\xi}+\lambda_1 A_{22}(\ )_{,\eta}$

其中

$$\bar{A}_{ij}=A_{ij}-\frac{8}{h^2}D_{ij}+\frac{16}{h^4}F_{ij}\qquad (i,j=4,5)$$

$$\bar{D}_{ij}=D_{ij}-\frac{8}{3h^2}F_{ij}+\frac{16}{9h^4}H_{ij}\quad (i,j=1,2,6)$$

$$\bar{F}_{ij}=F_{ij}=\frac{4}{3h^2}H_{ij}\qquad (i,j=1,2,6)\qquad (B)$$

$A_{ij},D_{ij},F_{ij},H_{ij}$是层合板的刚度系数；

$$(A_{ij},D_{ij},F_{ij},H_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,Z^2,Z^4,Z^6)dZ\qquad (i,j=1,2,6)$$

$$(A_{ij},D_{ij},F_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,Z^2,Z^4)dZ\qquad (i,j=4,5)\qquad (C)$$

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（编辑：徐维森）

NONLINEAR ANALYSIS OF LAMINATED ANGLE-PLY PLATE USING A HIGHER-ORDER SHEAR DEFORMATION THEORY

Liu Dong *Ou Maocai* *Liu Longquan*

(Department of Civil Engineering)

ABSTRACT A higher-order shear deformation theory is used to analyse nonlinear bending of symmetrically laminated angle-ply rectangular plates. The nonlinear governing differential equations are solved here by using perturbation method. Numerical rerults are presented graphically for vairous parameters of laminated plate. The present solutions are compared with those obtained using the classical plate theory. A lot of significient conclusions are obtained.

KEYWORDS Shear deformation, Nonlinear Bending, Laminated plate.