Acoustic design sensitivity analysis of structural sound radiation

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Abstract: This paper presents an acoustic design sensitivity (ADS) analysis on sound radiation of structures by using the boundary element method (BEM). We calculated the velocity distribution of the thin plate by analytical method and the surface sound pressure by Rayleigh integral, and expressed the sound radiation power of the structure in a positive definite quadratic form of the Hermitian with an impedance matrix. The ADS analysis of the plate was thus translated into the analysis of structure dynamic sensitivity and impedance matrix sensitivity. Finally, taking a simple supported thin plate as a simulation example, we investigated the relationships among the sound power, excitation frequency and thickness of the plate. The sound radiation power of the plate has a sharp rise in the vicinity of the plate natural frequency. The method proposed is verified to be able to predict the sound radiation of a thin plate with arbitrary boundary conditions and provide a theoretical basis for structural low noise design which is important to engineering design.

Keywords: acoustic design sensitivity analysis; sound radiation; impedance matrix; Rayleigh integral

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1 Introduction

Noise control is becoming more and more important to engineering design [1-5]. Although active vibration control can be used to decrease structural noise [6-8], low noise design is assume to be the most reliable mean of reducing radiated noise.

Based on the acoustic radiation modes [8-12] and further study on acoustic radiation, ADS (acoustic design sensitivity) analysis [13-20] was proposed to guide low noise design of structures. ADS analysis presents structural changes in sound radiation characteristics which can predict the structures with the minimization of sound radiation power. Weak radiator [18] can be acquired by ADS analysis, which is the key to low-noise structure design.

In ADS analysis of acoustic radiation, sound pressure is taken as the design objective [20], and the effects of design variables on sound pressure are studied. Nevertheless, sound pressure is spatially dependent and the calculation amount is very large. Sound power reveals itself as the most adequate mean for quantifying the radiation on the surface of a structure; it is related to structural characteristics and does not change with the spatial position. Therefore, sound power is more suitable than sound pressure for ADS analysis.

In this work, we studied the theoretical effects of changes in the thickness and excitation frequency on the sound radiation of a thin plate and focused on the ADS analysis of the plate with respect to sound radiation power. We assumed that air was the acoustic medium and a feedback coupling between the acoustic medium and the structure was a negligible factor.
Structural vibrations were excited with a time-harmonic mechanical loading of prescribed excitation frequency, amplitude, direction, and spatial distribution. The velocity distribution of structure was solved by analytical method and the surface sound pressure was computed by Rayleigh integral. Sound power was expressed in a positive definite quadratic form of the Hermitian based on the sound radiation mode theory. Then, ADS analysis of the thin plate with respect to sound radiation power was performed and transformed into the sensitivity of dynamic and impedance matrix with respect to design variables. Given a simple supported thin plate as a simulation example, the results verified the validity of the presented method.

2 Vibration analysis and sound radiation of thin plate

For a simple supported thin plate, the mode frequencies can be expressed as

$$\omega_{mn} = \frac{\pi}{a} \sqrt{\frac{m^2 + n^2}{a^2 + b^2}} \sqrt{\frac{D}{\rho h}},$$

where $m = 1, 2, \ldots$; $n = 1, 2, \ldots$; $D$, $\rho$ and $h$ are the flexural rigidity, density and thickness of the plate, respectively; and the dimensions of the plate are $a \times b$.

It is assumed that a time-harmonic concentrated loading of the prescribed amplitude $F_a$ and frequency $\omega$ is applied to the point $(\xi, \eta)$ of the plate. It can be expressed as

$$F(x, y, t) = F_a \delta(x - \xi)\delta(y - \eta)\sin\omega t,$$

where $x$ and $y$ are the space coordinates of loading; $t$ is time; and $\delta$ is the dirac delta function.

The displacement amplitude can be expressed as

$$X(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16F_a}{\rho h b} \frac{\sin(m\pi x/a)\sin(n\pi y/b)}{\omega^2 - \omega_0^2} \sin(m\pi x/a),$$

The time harmonic uniform loading of the prescribed amplitude $F_b$ and frequency $\omega$ can be expressed as

$$F(x, y, t) = F_b \sin\omega t.$$

The displacement amplitude can be expressed as

$$X(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16F_b}{\rho h mn b^2} \frac{\sin(m\pi x/a)\sin(n\pi y/b)}{\omega^2 - \omega_0^2} \sin(m\pi x/a).$$

The pressure at any observation point on the surface of the plate due to the plate normal surface velocity can be written as

$$p(P) = \frac{i \omega \rho}{2\pi} \int_{S} v_n(Q) e^{i r \omega} dS(Q),$$

where $r = \sqrt{\|\mathbf{Q} - \mathbf{P}\|$; $P$ is an observation point in the space; $Q$ is a point on the thin plate; $v_n$ is the plate normal surface velocity; $k$ is the wave number; and $S$ is the structural surface.

For an arbitrary shape structure, when the structural vibrations are excited by a time-harmonic mechanical loading of prescribed excitation frequency, the acoustic intensity at any point is written as

$$I(P) = \frac{1}{2} \text{Re}[p(P)v_n(P)],$$

where $*$ denotes the complex conjugate.

The total sound power radiated from the structure surface can be obtained by integrating the intensity over the structural surface area as

$$W = \frac{\omega \rho}{4\pi} \iint_{S} \int_{S} v_n(Q) e^{i r \omega}^{-1} v_n(P) dS(Q) dS(P).$$

In this work, four-node isoparametric elements were used in the mesh. The integral for the power was evaluated using Gaussian integral. The discretized sound power expression can be rewritten as

$$W = \frac{\omega \rho}{4\pi} \sum_{i=1}^{N_e} \sum_{j=1}^{N_e} J_i J_j v_i^* v_j e^{i r \omega},$$

where $J_i$ and $J_j$ are the values of the Jacobian matrixes; $v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4})$ denotes the node velocities of the $i$th element; $N_e$ is the shape function; and $H$ denotes the conjugation transpose.

With a notation for single point integration

$$z = \frac{\omega \rho}{4\pi} J_i J_j \sin kr e^{i r \omega},$$

where $J_i$ and $J_j$ are the values of the Jacobian matrixes; $v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4})$ denotes the node velocities of the $i$th element; $N_e$ is the shape function; and $H$ denotes the conjugation transpose.
the sound power can be rewritten in a matrix form as
\[ W = V^H Z V , \]  
where \( Z \) is the impedance matrix; and \( V \) is the velocity vector. The matrix \( Z \) is a Hermitian matrix. For an arbitrary \( V \), the sound power \( W \) is positive, and the matrix \( Z \) is positive definite.

### 3 ADS analysis with respect to design variable

The sensitivity of the sound power with respect to the design variable \( \psi \) is given by
\[ \frac{\partial W}{\partial \psi} = \partial V^H \frac{\partial Z}{\partial \psi} V + V^H \frac{\partial Z}{\partial \psi} V + V^H \frac{\partial V}{\partial \psi} Z V. \]  
(12)

Because the impedance matrix \( Z \) is a Hermitian matrix and the fact \( \frac{\partial V^H}{\partial \psi} = \left( \frac{\partial V}{\partial \psi} \right)^H \), Eq. (12) can be simplified as
\[ \frac{\partial W}{\partial \psi} = 2V^H \frac{\partial Z}{\partial \psi} V. \]  
(13)

Therefore, the sensitivity of the objective function is transformed into the sensitivity of a dynamic and impedance matrix, and only the calculation of \( \frac{\partial Z}{\partial \psi} \) is needed.

Calculating the partial derivative with respect to \( \psi \) of Eqs. (3) and (5) yields the sensitivity of dynamic.
\[ \frac{\partial V}{\partial \psi} = \omega \frac{\partial X}{\partial \psi}. \]  
(14)

The impedance matrix is a function of the excitation frequency. Hence, its sensitivity can be expressed by derivation to the design variable \( \psi \) as
\[ \frac{\partial Z}{\partial \psi} = \frac{\partial Z}{\partial \omega} \frac{\partial \omega}{\partial \psi}. \]  
(15)

Differentiate \( z \) given by Eq.(10) with respect to \( \omega \), and then assemble to give \( \frac{\partial Z}{\partial \omega} \). The first term can be computed easily. The second derivative \( \frac{\partial \omega}{\partial \psi} \) is the eigenvalue derivative. As the design changes, the resonant frequency of the structure also changes. Hence, the term \( \frac{\partial \omega}{\partial \psi} \) is not zero; it is the derivative of the eigenvalue with respect to thickness.

By the way, matrix \( Z \) is related to the structural shape and excitation loading only, and independent of the material, boundary condition and excitation position. If the design variable is not structural shape or excitation frequency, we can obtain
\[ \frac{\partial Z}{\partial \psi} = 0. \]  
(16)

The sensitivity of the sound power with respect to the design variable \( \psi \) is given by
\[ \frac{\partial W}{\partial \psi} = 2V^H \frac{\partial Z}{\partial \psi} V. \]  
(17)

### 4 Numerical examples

In the simulation tests, a simple supported rectangular plate was used. Its dimensions were \( a = 1000 \) mm, \( b = 1000 \) mm. The Young’s modulus of the plate was \( E = 2.1 \times 10^11 \) N/m\(^2\), Poisson’s ratio was \( \nu = 0.3 \), and the specific mass was \( \rho = 7800 \) kg/m\(^3\). The air density and the sound speed were \( \rho_0 = 1.225 \) kg/m\(^3\) and \( c = 343 \) m/s, respectively.

#### 4.1 Effects of design variables on sound radiation

In the first example, a time-harmonic concentrated loading was applied to the center of the plate; and in the second one, a time-harmonic uniform loading was applied.

##### 4.1.1 Excitation loading

The plate thickness was \( h = 3 \) mm. The excitation frequency \( f = (20 + 0.5n) \) Hz \((n = 0,1,2,...,98)\) was prescribed as the design variable. The natural frequency of the plate was obtained by finite element analysis, and the results are listed in Table 1.

Under a time-harmonic concentrated loading (Fig. 1), the plate had a high sound power when the excitation frequency of the loading approached the natural frequency of the plate, especially at the frequencies of 36.8Hz, 74.0Hz, and 96.2Hz. Because the thin plate was square, the \((m,n)\) order and the \((n,m)\) order were
the same. The sound radiation power was higher at these frequencies than at others.

![Fig. 1](image1.png)

**Fig. 1** Sound power as the function of frequency for a concentrated loading

Under a time-harmonic uniform loading (Fig. 2), the sound power was high at the frequencies of 14.8 Hz and 74.0 Hz. The displacement response of the \((m, n)\) order \((m=2k+1, n=2l+1)\) of the plate was zero. Although the excitation frequencies approached the natural frequencies at 36.8 Hz, 59.2 Hz, and 96.2 Hz, the sound power was very low.

![Fig. 2](image2.png)

**Fig. 2** Sound power as the function of frequency for a uniform loading

### Table 1: The first ten values of the natural frequency of the plate with a thickness of 3 mm

<table>
<thead>
<tr>
<th>No.</th>
<th>Frequency/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.8</td>
</tr>
<tr>
<td>2</td>
<td>36.8</td>
</tr>
<tr>
<td>3</td>
<td>36.8</td>
</tr>
<tr>
<td>4</td>
<td>59.2</td>
</tr>
<tr>
<td>5</td>
<td>74.0</td>
</tr>
<tr>
<td>6</td>
<td>74.0</td>
</tr>
<tr>
<td>7</td>
<td>96.2</td>
</tr>
<tr>
<td>8</td>
<td>96.2</td>
</tr>
<tr>
<td>9</td>
<td>125.8</td>
</tr>
<tr>
<td>10</td>
<td>125.8</td>
</tr>
</tbody>
</table>

### 4.1.2 Thickness

The excitation frequency was fixed at 100 Hz, and the thicknesses of the plate was the design variable given by \(h = (0.5 + 0.25n)\) mm \((n = 0, 1, 2, ..., 98)\). The plate natural frequency near the excitation frequency was obtained by finite element analysis, and the results are listed in Table 2.

It can be seen from Fig. 3 that plates with a thickness of 0.5 mm, 1 mm, 1.5 mm, 2 mm, 2.5 mm, 3 mm and 10.5 mm had high sound power because these plates had a natural frequency near the excitation frequency.

![Fig. 3](image3.png)

**Fig. 3** The natural frequency of the plate with different thicknesses

Table 2 illustrates that sound power was high at the thickness of 0.5 mm, 2 mm and 10.5 mm because only the \((m, n)\) order \((m=2k+1, n=2l+1)\) affected the sound power of the plate. When the change in the thickness of a plate made its natural frequency approach the excitation frequency, the sound power dramatically increased.

### Table 2: Natural frequencies near the excitation frequency for plates of different thickness

<table>
<thead>
<tr>
<th>Thickness/mm</th>
<th>Frequency/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>98.1</td>
</tr>
<tr>
<td>1.0</td>
<td>98.3</td>
</tr>
<tr>
<td>2.0</td>
<td>98.2</td>
</tr>
<tr>
<td>3.0</td>
<td>98.5</td>
</tr>
<tr>
<td>4.0</td>
<td>98.8</td>
</tr>
<tr>
<td>10.5</td>
<td>100.8</td>
</tr>
</tbody>
</table>
A notation $\Delta \omega$ was introduced to denote the difference between the plate natural frequency and the excitation frequency. The sound power was a decreasing function of the difference ($\Delta \omega$). When $\Delta \omega$ decreased the sound power increased sharply (Figs. 3 and 4).

With the plate thickness increasing, the stiffness increased and the frequency consequently increased also, and there was generally a surge of sound radiated power. The graph of power vs thickness had peaks when the plate resonance frequency was near the excitation frequency and then showed a steady decline as the thickness continued to increase, as can be seen in Figs. 3 and 4.

4.2 ADS analysis of thin plate

4.2.1 Frequency

Fig. 5 illustrates that the ADS analysis on frequency. A time-harmonic concentrated loading with prescribed amplitude $F = 1$ N was applied to the point (250 mm, 150 mm). The plate thickness was 3 mm. The excitation frequency was prescribed as the design variable.

It can be seen that the plate had high sound power and high sensitivity when the excitation frequency approached the plate natural frequency.

4.2.2 Thickness

The ADS analysis on thickness is showed in Fig. 6. A time-harmonic concentrated loading with a prescribed frequency of 100 Hz and amplitude $F = 1$ N was applied to the point (250 mm, 150 mm). The plate thickness was adopted as the design variable.

Plates of a natural frequency near the excitation frequency had high sound power and high sensitivity, implying that sound power was changed significantly by modification of structural parameters. This suggests an effective approach to structure dynamic modification.

5 Conclusion

We used analytical method and Rayleigh integral method to study the sound radiation of thin plates. Taking a simple supported thin plate as a simulation example, we conducted the ADS analysis on the design variables of the plate. Simulation results showed that changes in the frequency and thickness of the plate have a greater impact than other factors on sound radiation. The sound radiation power does not reduce as the thickness increases in a certain frequency range and has a sharp increase in the vicinity of the natural frequency of a structure. The method we proposed in this paper can predict the sound radiation of a thin plate.
with arbitrary boundary conditions and provide a theoretical basis for structural low noise design which is important to engineering design.

A plate has high sound power and high sensitivity when its natural frequency approaches the excitation frequency.

ADS analysis indicated that modifying structural parameters significantly changes sound power, providing an effective scheme for structure dynamic modification.

Fig. 5 Sound power and ADS (acoustic design sensitivity) as a function of frequency, where \( \log_{10}|\text{ADS}| \) is the decimal-based logarithm of ADS absolute value

Fig. 6 Sound power and ADS (acoustic design sensitivity) as a function of thickness, where \( \log_{10}|\text{ADS}| \) is the decimal-based logarithm of ADS absolute value

References


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