A method for measuring investment risk based on a relative risk - excess revenue model

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Abstract: This paper identifies the shortcomings of variance and semi-variance methods in investment risk measurement and introduces a new model, namely RR-ER (relative risk and excess revenue) model, which takes account of the revenue over expectation problem. Properties of RR-ER model and the consistency between RR-ER model and traditional risk measure model with regard to continuous random variables are discussed. Case analysis is presented to prove the practicality and efficiency of this new method.

Keywords: relative risk; excess revenue; case analysis

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1 Introduction

With the development of investment, how to find a proper method to measure risk becomes a popular topic in the financial area. Risk changes are becoming more and more complicated. So the accuracy of risk measurement will affect investment decisions and final revenue. Therefore, it is of great significance to find an effective risk measuring method in the project investment.

In 1952, Markowitz [1] first proposed that variance of the historical data of return rate for investment could be used as a measure of investment risk. And he simplified the problem for portfolio investment decision to a two-dimensional problem between risk and profit. It has become the core of modern financial theory because of its practicality, simplicity and good statistic property of variance (especially under the condition that the return rate obeys the normal distribution).

However, with the progress of investment research, people realize that using variance to measure risk of investment has many disadvantages. For example, risk computation with the variance method provides results above and below the average which are against the real feel of investors. The part above the average is profit, not risk. The investors are only concerned with the part below the average, and this is the real risk. So the variance method exaggerates the risk. If we make investment decisions according to the risk minimization rule, the variance method will make them lose many opportunities to receive more revenue. So researchers put forward many new methods, such as semi-variance method, logarithmic utility method, and VaR (value at risk) method. VaR method computes the absolute value of risk, so it is simpler than variance method. Moreover, VaR method mainly cares about the maximum of losing. It is more scientific than variance method, so it is more popular. But when we compute VaR by parameter method, it is subjective to select a model and make an assumption of distribution. These are dependent on experience and bring some difficulties in computation. So we propose a new method on the basis of variance method, semi-
variance method and VaR method. Our method can not only measure the relative risk of investment, but also give enough consideration to revenue over expectation. We call it RR-ER model, short for relative risk and excess revenue method.

2 RR-ER model

Based on the studies of Stone [2] and Fishburn [3], which use semi-variance instead of variance to measure risk, we define the left-side semi-variance and right-side semi-variance as follows.

\[
\sigma^2_+ = \int_{-\infty}^{x_\mu} (\mu - x)^2 f(x)dx,
\]

\[
\sigma^2_- = \int_{x_\mu}^{\infty} (\mu - x)^2 f(x)dx.
\]

Obviously,

\[
\sigma^2_+ + \sigma^2_- = \int_{-\infty}^{x_\mu} (\mu - x)^2 f(x)dx + \int_{x_\mu}^{\infty} (\mu - x)^2 f(x)dx = \int_{-\infty}^{\infty} (\mu - x)^2 f(x)dx = \sigma^2.
\]

In the above equations, \(\sigma\) is standard deviation; \(\mu\) is mean value; \(x\) is the value of random variable \(X\); and \(f\) is density function of continuous random variable \(X\).

We consider the risk and the profit, respectively, that is the part when random variable \(X \leq \mu\) and the part when random variable \(X > \mu\). So the risk part and the profit part do not offset each other, and we do not need to use square or absolute value operation to measure risk and profit.

In the following, we consider the risk part when random variable \(X \leq \mu\) and the part when random variable \(X > \mu\) respectively, \(\mu\) is usually the average and is generalized to a fixed target value or constant. The RR-ER model is defined as follows.

2.1 Relative risk and excess profit of discrete random variable

**Definition 1.** Suppose the distribution of discrete random variable \(X\) is:

\[ P\{X = x_k\} = p_k, \quad k = 1, 2, 3, \ldots \]

If the series \(\sum_{x_\eta \leq \eta} (\eta - X_i)p_i\) converges, where \(\eta\) is a fixed level, then we call it the relative risk of random variable \(X\), denoted as \(RR(\eta, X)\), that is

\[
RR(\eta, X) = \sum_{x_\eta \leq \eta} (\eta - X_i)p_i. \tag{1}
\]

Similarly, if series \(\sum_{x_\eta > \eta} (X_i - \eta)p_i\) converges, then we call it the excess profit of random variable \(X\), denoted as \(ER(\eta, X)\), that is

\[
ER(\eta, X) = \sum_{x_\eta > \eta} (X_i - \eta)p_i, \tag{2}
\]

where \(X\) represents the profit of investment, and usually we set \(\eta = E(X) = \mu\), that is the expected value of investment.

We note that when we compute \(RR(\eta, X), x_i \leq \eta, \eta - x_i \geq 0, p_i \geq 0\), so

\[
RR(\eta, X) = \sum_{x_\eta \leq \eta} (\eta - X_i)p_i \geq 0.
\]

Similarly,

\[
ER(\eta, X) = \sum_{x_\eta > \eta} (X_i - \eta)p_i \geq 0.
\]

2.2 Relative risk and excess profit of continuous random variable

**Definition 2.** Suppose the density function of continuous random variable \(X\) is \(f(x)\), and the integrals

\[
\int_{-\infty}^{\eta} (\eta - x)f(x)dx \quad \text{and} \quad \int_{\eta}^{\infty} (x - \eta)f(x)dx
\]

converge, then we call the integrals above the relative risk and excess profit of random variable \(X\), denoted as \(RR(\eta, X)\) and \(ER(\eta, X)\), that is

\[
RR(\eta, X) = \int_{-\infty}^{\eta} (\eta - x)f(x)dx, \tag{3}
\]

\[
ER(\eta, X) = \int_{\eta}^{\infty} (x - \eta)f(x)dx. \tag{4}
\]

Usually we set \(\eta = E(X) = \mu\), that is the expected value of investment.

For continuous random variable \(X\), the properties of \(RR(\eta, X)\) and \(ER(\eta, X)\) will be introduced in Section 4.

2.3 Risk coefficient and efficiency coefficient in risk investment

As for projects of risk investment, investors usually consider both risk and profit. In most assessment of
investment projects, investors should balance the risk and profit. When the chance of getting revenue is larger than that of incurring losses, we can consider making this investment. When the chances are equal or the chance of losses is larger, whether to make the investment or not also depends on the attitudes to risks and abilities to bear risks. Risk aversion investors will avoid any risk. On the contrary, risk preference investors will prefer the project with larger risk that potentially comes with a larger revenue. But all investors should know exactly the results of their decisions.

To give a better guidance to investors, we introduce the definitions of risk coefficient and efficiency coefficient.

**Definition 3.** Suppose random variable $X$ satisfies

$$\alpha(\eta, X) = \frac{\text{RR}(\eta, X)}{\text{RR}(\eta, X) + \text{ER}(\eta, X)}.$$  

Then we call $\alpha$ the risk coefficient of a fixed profit target $\eta$. From the definition we can see that risk coefficient $0 \leq \alpha \leq 1$. The closer to 0 $\alpha$ is, the smaller the risk is, and the closer to 1 $\alpha$ is, the larger the risk is. Especially, when $\alpha = 0$, the risk of investment is 0, that is risk free investment, e.g. government bonds and bank deposits.

The profit coefficient is defined similarly as follows.

**Definition 4.** Suppose random variable $X$ satisfies

$$\beta(\eta, X) = \frac{\text{ER}(\eta, X)}{\text{RR}(\eta, X) + \text{ER}(\eta, X)}.$$  

Then we called $\beta$ the efficiency coefficient of a fixed profit target $\eta$. The efficiency coefficient $\beta$ satisfies $0 \leq \beta \leq 1$. The closer to 0 $\beta$ is, the smaller the profit is, and the closer to 1 $\beta$ is, the larger the profit is.

Obviously, the sum of risk coefficient and efficiency coefficient is 1, that is

$$\alpha(\eta, X) + \beta(\eta, X) = 1.$$  

Generally,

1) when $\beta(r) > 0.5$, consider investment;
2) when $\beta(r) < 0.5$, consider dropping investment;
3) when $\beta(r) = 0.5$, whether to invest or not depends on the degree of risk preference and ability to bear risk. Risk aversion investors drop investment and risk preference investors can consider investment.

### 3 Steps of RR-ER model

#### 3.1 Preliminary knowledge-data integration technology

**Definition 5.** Suppose $[a,b]$ is an interval number $f_r((a,b)) = \int_a^b \frac{\tau(0)(b-y)+\tau(y)(b-a)}{b-a} dy$, where $\tau: [0,1] \rightarrow [0,1]$ is a function satisfying

1) $\tau(0) = 0$;
2) $\tau(1) = 1$; and
3) if $x > y$, then $\tau(x) \geq \tau(y)$.

Then we call $f$ the data integration operator on a continuous interval, and $\tau$ the basic unit-interval monotonic function, that is BUM (basic unit-interval monotonic) function. So the operator has the following property:

**Theorem 1 (Boundedness).** As for any BUM function $\tau$, we have $a \leq f_r([a,b]) \leq b$.

For example, if $\tau(x) = x$, where $r \geq 0$ is a constant, then

$$f_r([a,b]) = \frac{b + ra}{r+1}.$$  

#### 3.2 Case analysis

Nowadays, the major problem in Chinese stock market is that retail investors are the majority and they lack rational consideration. They do not obey the “analysis-select-portfolio” procedure and just follow the majority. This makes the capital unable to attain a optimum distribution. To improve this condition, Shanghai stock market needs rational investors and funds to join in and concentrate the diversified investment to make rational and effective investment. Then our stock market can optimize the allocation of resources. [23]

There are many investors in China whose portfolios have seventy kinds of stocks or even more. These excess portfolios eliminate part of the risk, but the profit is not satisfied for two reasons: 1) the administrative expense is high; and 2) these excess portfolios may have some bad stocks which have high risks and low revenue. To improve this situation we should make effective selections before decisions.
Hereinafter we give some examples to explain this.
We select four representative stocks and list their historical data in Table 1 below.

If the expected rate of return is $\eta = 10\%$, which stock has the minimum relative risk and maximum excess profit? Give the order of these four stocks according to their RR-ER index.

**Step 1.** Turn the interval number of the stock to an effective number by using data integration technology.

As for the K-line graph of stock A, let $r = 2$, and use Eq. (8):

$$
x_1 = f_{r,2}([19,27]) = 21.67;
$$

$$
x_2 = f_{r,2}([4,12]) = 6.67;
$$

$$
x_3 = f_{r,2}([-28,-24]) = -26.67.
$$

As for stock B, let $r = \frac{1}{3}$, and use Eq. (8):

$$
x_1 = f_{r,\frac{1}{3}}([24,32]) = 30.00;
$$

$$
x_2 = f_{r,\frac{1}{3}}([6,10]) = 9.00;
$$

$$
x_3 = f_{r,\frac{1}{3}}([-35,-27]) = -29.00.
$$

As for stock C, let $r = \frac{1}{2}$, and use Eq. (8):

$$
x_1 = f_{r,\frac{1}{2}}([0.4,1.2]) = 0.93;
$$

$$
x_2 = f_{r,\frac{1}{2}}([-8,-4]) = -5.33;
$$

$$
x_3 = f_{r,\frac{1}{2}}([32,36]) = 34.67.
$$

As for stock D, let $r = 1$, and use Eq. (8):

$$
x_1 = f_{r,1}([14,16]) = 15.00;
$$

$$
x_2 = f_{r,1}([-8,-4]) = -6.00;
$$

$$
x_3 = f_{r,1}([20,23]) = 21.50.
$$

**Step 2.** Compute RR and ER of these four stocks with Eqs. (1) and (2), respectively, setting $\eta = 10\%$.

$$
RR(r_1) = \sum_{x_i \in \mathcal{X}} (\eta - X_i)p_i = (10 - 6.67) \times 0.3 + [10 - (-26.67)] \times 0.2 = 9.33;
$$

$$
RR(r_2) = \sum_{x_i \in \mathcal{X}} (\eta - X_i)p_i = (10 - 9.00) \times 0.3 + [10 - (-29.00)] \times 0.2 = 8.10;
$$

$$
RR(r_3) = \sum_{x_i \in \mathcal{X}} (\eta - X_i)p_i = (10 - 0.93) \times 0.5 + [10 - (-5.33)] \times 0.2 = 7.60;
$$

$$
RR(r_4) = \sum_{x_i \in \mathcal{X}} (\eta - X_i)p_i = (10 - (-6.00)) \times 0.3 = 4.80.
$$

So,

$$
RR(r_2) < RR(r_3) < RR(r_1) < RR(r_4).
$$

When the expected rate of return is $10\%$, RR of stock A is the largest, and RR of stock D is the smallest.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Return rate ($%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal real estate market</td>
</tr>
<tr>
<td></td>
<td>Bull market ($P=0.5$)</td>
</tr>
<tr>
<td>Stock A ($r_1$)</td>
<td>19.0 to 27.0</td>
</tr>
<tr>
<td>Stock B ($r_2$)</td>
<td>24.0 to 32.0</td>
</tr>
<tr>
<td>Stock C ($r_3$)</td>
<td>0.4 to 1.2</td>
</tr>
<tr>
<td>Stock D ($r_4$)</td>
<td>4.0 to 8.0</td>
</tr>
</tbody>
</table>
ER\left(r_1\right) = \sum_{\eta > \eta} (X_i - \eta)p_i = (21.67 - 10) \times 0.5 = 5.84;
ER\left(r_2\right) = \sum_{\eta > \eta} (X_i - \eta)p_i = (30.00 - 10) \times 0.5 = 10.00;
ER\left(r_3\right) = \sum_{\eta > \eta} (X_i - \eta)p_i = (34.67 - 10) \times 0.2 = 4.93;
ER\left(r_4\right) = \sum_{\eta > \eta} (X_i - \eta)p_i = (15.00 - 10) \times 0.5 + (21.500 - 10) \times 0.2 = 4.80.

So,
ER\left(r_2\right) < ER\left(r_3\right) < ER\left(r_4\right) < ER\left(r_1\right).

When the expected rate of revenue is 10%, ER of stock B is the largest and ER of stock D is the smallest.

**Step 3.** Compute the risk coefficient \( \alpha \) of these four stocks with Wq. (5).

\[ \alpha\left(r_1\right) = \frac{RR\left(r_1\right)}{RR\left(r_1\right) + EP\left(r_1\right)} = \frac{9.33}{9.33+5.84} = 0.62; \]
\[ \alpha\left(r_2\right) = \frac{RR\left(r_2\right)}{RR\left(r_2\right) + EP\left(r_2\right)} = \frac{8.10}{8.10+10.00} = 0.45; \]
\[ \alpha\left(r_3\right) = \frac{RR\left(r_3\right)}{RR\left(r_3\right) + EP\left(r_3\right)} = \frac{7.60}{7.60+4.93} = 0.61; \]
\[ \alpha\left(r_4\right) = \frac{RR\left(r_4\right)}{RR\left(r_4\right) + EP\left(r_4\right)} = \frac{4.80}{4.80+4.80} = 0.50. \]

So,
\[ \alpha\left(r_2\right) < \alpha\left(r_3\right) < \alpha\left(r_4\right) < \alpha\left(r_1\right). \]

From the point of risk coefficient, the risk of stock B is the smallest and the risk of stock A is the largest.

**Step 4.** Compute the efficiency coefficient \( \beta \) of these four stocks with Eq. (7).

\[ \beta\left(r_1\right) = 1 - \alpha_1 = 1 - 0.62 = 0.38; \]
\[ \beta\left(r_2\right) = 1 - \alpha_2 = 1 - 0.45 = 0.55; \]
\[ \beta\left(r_3\right) = 1 - \alpha_3 = 1 - 0.61 = 0.39; \]
\[ \beta\left(r_4\right) = 1 - \alpha_4 = 1 - 0.50 = 0.50. \]

So,
\[ \beta\left(r_2\right) > \beta\left(r_3\right) > \beta\left(r_4\right) > \beta\left(r_1\right). \]

From the point of efficiency coefficient, the profit of stock B is the largest and the profit of stock A is the smallest.

**Step 5.** Give the order of these four stocks according to the analysis above, and make rational investment strategy.

According to the analysis above, when the profit target is 10%,

\[ r_2 > r_4 > r_3 > r_1. \]

Some advice:
1) The risk coefficients of stock A and C are big and the efficiency coefficients of them are small. Moreover, \( \beta\left(r_1\right) = 0.38 < \beta\left(r_3\right) = 0.39 < 0.5. \)

So we should not invest on stock A and C.
2) Stock B has the most profit in these four stocks and the efficiency coefficient is larger than 0.5, which is \( \beta\left(r_2\right) = 0.55 > 0.5. \)

So we can invest on stock B.
3) The efficiency coefficient of stock D is \( \beta\left(r_4\right) = 0.50, \) whether to invest or not depends on the degree of risk preference and ability to bear risk. But in this example, we can see that there are negative correlations between stock B and D. If we make investment portfolio on them, we can not only receive revenue, but also have the hedge function.

**4 Theoretical significance of RR-ER model**

Now we study the properties of the RR-ER model and the consistency between RR-ER model and traditional risk measure model.

Suppose \( X \) is a continuous random variable, \( F(x) \) and \( f(x) \) are the distribution function and density function of \( X \) respectively, and \( \mu \) is the expected value of \( X \).

Then, \( RR(\eta, X) = \int_{-\infty}^{\eta} (x - x)f(x)dx \) and \( ER(\eta, X) = \int_{\eta}^{\infty} (x - \eta)f(x)dx \) have the following properties:
1) \( RR(\eta, X) \) is a continuous, nonnegative nondecreasing and convex function. \( ER(\eta, X) \) is a continuous, nonnegative nonincreasing and convex
function.

From the definition of continuous random variable we can see that $\text{RR}(\eta, X)$ and $\text{ER}(\eta, X)$ are continuous. Now we only need to prove their monotonicity and convexity.

Proof.

$$\frac{d}{d\eta} \left[ \text{RR}(\eta, X) \right] = \frac{d}{d\eta} \int_{-\infty}^{\eta} (x - \eta) f(x) dx =$$

$$\frac{d}{d\eta} \left[ \eta \int_{-\infty}^{\eta} f(x) dx - \int_{-\infty}^{\eta} x f(x) dx \right] =$$

$$\int_{-\infty}^{\eta} f(x) dx \geq 0.$$

Similarly,

$$\frac{d}{d\eta} \left[ \text{ER}(\eta, X) \right] = -\int_{\eta}^{\infty} f(x) dx \leq 0.$$

So, $\text{RR}(\eta, X)$ is a continuous and nondecreasing function of $\eta$. $\text{ER}(\eta, X)$ is a continuous and nonincreasing function.

$$\frac{d^2}{d\eta^2} \text{RR}(\eta, X) = \frac{d}{d\eta} \int_{-\infty}^{\eta} f(x) dx = f(\eta) > 0;$$

$$\frac{d^2}{d\eta^2} \text{ER}(\eta, X) = \frac{d}{d\eta} \left[ -\int_{\eta}^{\infty} f(x) dx \right] = f(\eta) > 0.$$

So, $\text{RR}(\eta, X)$ and $\text{ER}(\eta, X)$ are convex functions.

2) If $\text{RR}(\eta_0, X) > 0$, then for all $\eta \geq \eta_0$, $\text{RR}(\eta, X)$ is increasing (we can see this from the geometric interpretation of integral).

These properties tell us the risk is larger when the profit target is higher.

$$\text{RR}(\eta, X) = \int_{-\infty}^{\eta} (x - \eta) f(x) dx = \int_{-\infty}^{\eta} F(x) dx = P\{X \leq \eta\} E[\eta - x \mid X \leq \eta].$$

Proof.

$$\text{RR}(\eta, X) = \int_{-\infty}^{\eta} (x - \eta) f(x) dx = \int_{-\infty}^{\eta} (x - \eta) dF(x)$$

$$= (\eta - x) F(x) \big|_{x=\eta} + \int_{-\infty}^{\eta} F(x) dx = \int_{-\infty}^{\eta} F(x) dx;$$

and

$$P\{X \leq \eta\} E[\eta - x \mid X \leq \eta] =$$

$$P\{X \leq \eta\} \int_{-\infty}^{\eta} (x - \eta) dF(x) =$$

$$P\{X \leq \eta\} \int_{-\infty}^{\eta} (x - \eta) \frac{dF(x)}{F(\eta)} =$$

$$P\{X \leq \eta\} \frac{1}{F(\eta)} \int_{-\infty}^{\eta} (\eta - x) dF(x) =$$

$$P\{X \leq \eta\} \frac{\text{RR}(\eta, X)}{P\{X \leq \eta\}} = \text{RR}(\eta, X).$$

So,

$$\text{RR}(\eta, X) = \int_{-\infty}^{\eta} (\eta - x) f(x) dx =$$

$$\int_{-\infty}^{\eta} F(x) dx = P\{X \leq \eta\} E[\eta - x \mid X \leq \eta].$$

(10)

$$\text{RR}(\eta, X) - \text{EP}(\eta, X) = \eta - \mu.$$

Proof.

$$\text{RR}(\eta, X) - \text{EP}(\eta, X) =$$

$$\int_{-\infty}^{\eta} (\eta - x) f(x) dx - \int_{-\infty}^{\eta} (\eta - \eta) f(x) dx =$$

$$\int_{-\infty}^{\eta} \eta f(x) dx - \int_{-\infty}^{\eta} x f(x) dx = \eta - \mu.$$

The left side of the equation is the difference between relative risk and excess profit. The right side of the equation is the difference between the mean value and the fixed profit target. This is a constant. This property tells that in investment, the larger the risk is, the more the profit you will get, and the difference is a constant. Just as people said, “high risk, high payoff” and “there is no such thing as a free lunch”. But there are still some techniques we can learn. Our RR-ER method cannot make the most profit under the smallest risk, but we give a good profit point considering the ability to bear risk.

$$\text{ER}(\eta, X) = \int_{\eta}^{\infty} (x - x) f(x) dx = \int_{\eta}^{\infty} F(x) dx - (\eta - \mu) =$$

$$P\{X \geq \eta\} E[x - \eta \mid X \geq \eta].$$

(11)

$$\text{RR}(\eta, X) - \text{ER}(\eta, X) = \eta - \mu.$$

So,

$$\text{ER}(\eta, X) = \text{RR}(\eta, X) - (\eta - \mu) = \int_{-\infty}^{\eta} F(x) dx - (\eta - \mu).$$
Moreover,
\[ P[X \geq \eta] = \int_{\eta}^{\infty} (x-\eta) f(x) \, dx = \frac{P[X \geq \eta]}{E[X]} (x-\eta) f(x) \, dx = \int_{\eta}^{\infty} (x-\eta) \, dx. \]

So,
\[ \text{ER}(\eta, X) = \int_{-\infty}^{\eta} (x-\eta) f(x) \, dx = \int_{-\infty}^{\eta} f(x) \, dx - \eta \mu. \]

From the property of improper integral, \( \lim_{\eta \to -\infty} \text{ER}(\eta, X) = 0 \) and \( \lim_{\eta \to +\infty} \text{ER}(\eta, X) = 0 \).

From Eqs. (10) and (12),
\[ \lim_{\eta \to -\infty} [\text{RR}(\eta, X) - \eta \mu] = 0. \]

From Eqs. (12) and (13), we can see that the axis \( \eta \) and line \( \eta \mu - \eta \) are two asymptotes of curve \( \text{RR}(\eta, X) \).

From Eqs. (1) to (7), we can see that \( \text{RR}(\eta, X) \) is a good index to measure risk. If \( \eta \) is the horizontal axis and \( \text{RR}(\eta, X) \) is the vertical axis, then we obtain a curve \( (\eta, \text{RR}(\eta, X)) \). This curve has two asymptotes: axis \( \eta \) and line \( \eta \mu - \eta \) as shown in Fig. 1. The space between curve \( (\eta, \text{RR}(\eta, X)) \) and its asymptotes denotes the dispersion degree between the investment profit and the expected value \( \mu \).

The shaded part \( S_1 \) in Fig. 1 denotes the risk when \( \eta \leq \mu \), that is
\[ S_1 = \int_{-\infty}^{\mu} \text{RR}(\eta, X) \, d\eta = \int_{-\infty}^{\mu} \left[ \int_{-\infty}^{\eta} (x-\eta) f(x) \, dx \right] d\eta = \int_{-\infty}^{\mu} (\eta - \mu) f(x) \, dx + \int_{-\infty}^{\mu} (\mu - x) f(x) \, dx = \frac{1}{2} \int_{-\infty}^{\mu} (\mu - x)^2 f(x) \, dx. \]

From the property of semi-variance:
\[ \int_{-\infty}^{\mu} (\mu - x)^2 f(x) \, dx + \frac{1}{2} \int_{-\infty}^{\mu} (\mu - x)^2 f(x) \, dx = \sigma^2. \]

We note that the risk in RR-ER model is half of that in semi-variance method.

From the property of semi-variance:
\[ \sigma^2 = 2 \left[ \int_{-\infty}^{\mu} \text{RR}(\eta, X) \, d\eta + \int_{\mu}^{\infty} \text{ER}(\eta, X) \, d\eta \right]. \]

This property tells that the space between curve \( (\eta, \text{RR}(\eta, X)) \) and its asymptotes equals half of the variance.

We introduce the relative risk function and excess profit function to help us make investment decision by using property (4). Considering the degree of risk preference, we add a risk preference coefficient \( \lambda \), \( \lambda \in (0,1) \). So,
\[ F(\lambda, \eta) = (1 - \lambda) \cdot \text{RR}(\eta, X) - \lambda \cdot \text{ER}(\eta, X). \]

We will discuss \( F(\lambda, \eta) \) for different \( \lambda \). When
$F(\lambda, \eta) > 0$, the profit may be larger than the loss given a certain risk preference. When $F(\lambda, \eta) < 0$, the profit may be smaller than the loss.

5 Application values of RR-ER model

Let $X$ be the revenue value of the investment project. Then $X$ should be a set $\{X_1, X_2, X_3, \ldots, X_n\}$, where the expected minimum revenue is $X_{\min}$, the expected maximum is $X_{\max}$, and the expected revenue is $\mu$.

For the convenience of computation and observation, we draw a dotted line $BE$ and approximately use trapezoid $ABEF$ instead of risk part $S_1$ and triangle $BCE$ instead of excess revenue part $S_2$, as is shown in Fig. 2.

$$S_1 \approx S_{ABEF} = \frac{1}{2}[RR(X_{\min}) + RR(\mu)](\mu - X_{\min}). \quad (16)$$

$$S_2 \approx S_{BCE} = \frac{1}{2}RR(\mu)(X_{\max} - \mu). \quad (17)$$

It is easy to notice that $X_{\min} = 5$, $X_{\max} = 15$, and $\mu = E(X) = 10$. Substituting them into Eq. (3) leads to

$$RR(X_{\min}) = RR(5) = \int_{-\infty}^{\eta} (\eta - x)f(x)dx = \int_{-\infty}^{5} (5 - x)\times 0dx = 0;$$

$$RR(\mu) = RR(10) = \int_{5}^{10} (10 - x)\times \frac{1}{10}dx = 1.25.$$

Then, substitute the results above into Eqs. (16) and (17):

$$S_1 \approx \frac{1}{2}[RR(5) + RR(10)](10 - 5) = 3.13;$$

$$S_2 \approx \frac{1}{2}RR(10)\times (15 - 10) = 3.13.$$

From Eq. (5) we can obtain the risk coefficient

$$\alpha_1 = \frac{s_1}{s_1 + s_2} = 0.5.$$

Option 2. The rate of revenue of the investment obeys exponential distribution and the density function is

$$f(x) = \begin{cases} \frac{1}{10}e^{-\frac{x}{10}}, & x > 0; \\ 0, & x \leq 0. \end{cases}$$

It is easy to notice that $X_{\min} = 5$, $X_{\max} = 20$, and $\mu = E(X) = 10$. Substitute them into Eq. (3):

$$RR(X_{\min}) = RR(5) = \int_{-\infty}^{\eta} (\eta - x)f(x)dx = \int_{0}^{5} (5 - x)\times \frac{1}{10}e^{-\frac{x}{10}}dx = 1.04;$$

$$RR(\mu) = RR(10) = \int_{5}^{10} (10 - x)\times \frac{1}{10}e^{-\frac{x}{10}}dx = 3.68.$$

Then, substitute the results above into Eqs. (16) and (17):

$$S_1 \approx \frac{1}{2}[RR(5) + RR(10)](10 - 5) = 11.80;$$
From Eq. (5) we can obtain the risk coefficient 

\[
\alpha = \frac{S_1}{S_1 + S_2} = 0.39.
\]

Compare the risk coefficients in two options: \(\alpha_1 > \alpha_2 = 0.39\). That means if the expected revenue in Option 1 equals to the expected revenue in Option 2, the probability to obtain excess revenue in Option 2 (61%) is higher than that in Option 1 (50%), and the expected maximum revenue in option 2 \((X_{\text{max}} = 20)\) is also higher than that in Option 1 \((X_{\text{max}} = 15)\). So Option 2 is better than Option 1 under this condition.

Notice that in the process of approximate computation, we replace the curve with a straight line. So there must be some approximating errors. In order to reduce the errors, further studies will be needed.

6 Conclusions

RR-ER model is consistent with the traditional risk measure model and semi-variance model. But as a risk management tool, it has unique advantages. First, making investment has something to do with risk preference. Investors not only care about the risk, but also the excess profit. They will balance the profit and the risk, and then make a decision. The RR-ER model helps them make such decisions. Second, our method does not need "extraction of root" operation or "square" operation. So it is convenient to compute. Third, because the stock market transactions are in a random environment. Investors cannot acquire the accurate probability distribution of every stock but an interval estimate. In the case analysis part, we give a solution to this situation. First, we use data integration technique to turn interval numbers of stock revenue into effective numbers, and then use RR-ER model to select stocks according to their relative risks and excess revenue. The steps are very clear, so it is easy to develop computation software. Finally, the disadvantage of our model is that we only discuss one single stock. As for investment portfolio, it still need further research.

References


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