Calculating method of contact stress for non-circular gears

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Abstract: According to Hertz theory, the difference of contact stress for non-circular gears and equivalent gears is compared in the paper, a calculating method of contact stress for non-circular gears by using equivalent gears is researched, and computing formulas of power and rotation speed for equivalent gears are deduced. A numerical simulation of contact stress for non-circular gears has also been conducted based on the finite element method. By the comparison of fitting curves, the feasibility of using equivalent gears instead of non-circular gears to calculate the contact stress is testified.

Keywords: non-circular gear; contact stress; equivalent gear; calculating method

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1 Introduction

Non-circular gears are a mechanism that can achieve a variable transmission ratio. In the past, because of design complexity and machining difficulty, its application is limited in a few fields [1-3]. With the rapid development of computer and CNC (computer numerical control) technology in recent years, more and more applications appeared. However, there are still very few studies on the fatigue strength of non-circular gears. Because of the lack of calculating method for checking the fatigue strength, its applications have been limited again in a highly reliable mechanism. Pitting is a common failure mode in gears. It can cause many spots on the tooth surface because of the metal fatigue flaking, when the contact stress on the tooth surface is larger than the fatigue limit stress of the material. To make gears work effectively in the life, the tooth should have enough anti-pitting capacity. According to the FEM (finite element method) and the calculating method of load capacity for spur gearing, the contact stress of non-circular gears is discussed in the paper. Especially, the calculating method of load capacity for checking the fatigue strength of non-circular gears is further perfected.

2 Calculation model of contact stress for gear tooth

The calculation of contact stress for spur gears is based on the Hertz’s elastic contact theory between two
cylinders \[2\]. As shown in Fig. 1, there are two cylinders, whose radii are separately \(r_1\) and \(r_2\); \(L\) is the length; \(\rho\) is the relative radius of curvature of two cylinders; moduli of elasticity are \(E_1\) and \(E_2\); and Poisson’s ratios for materials are \(v_1\) and \(v_2\), respectively. Before loading a force, it is a line contact on the cylinders. Under loading the force \(F\), it changes to an area contact. The distribution of contact stress follows an ellipse rule on the contact area. The calculating formula of the maximum contact stress is

\[
\sigma_{\text{Hmax}} = \frac{F}{\pi L} \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \left( \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \right)
\]

(1)

![Fig. 1 The analysis model of contact stress](image)

**3 Feasibility analysis of using equivalent gears instead of non-circular gears to calculate the contact stress**

In spur gearing, it is segmented into a single teeth-meshing area and a double teeth-meshing area on the tooth profile \[3\]. Because there are two pairs of teeth meshing simultaneously on the double teeth-meshing area, the force is loaded on two teeth. So the contact stress of tooth profiles is smaller than that on a single teeth-meshing area. On the single teeth-meshing area, the normal force is equal on all meshing points, but the relative radius of curvature of two tooth profiles is the shortest on the critical point of the single teeth-meshing area. So it can be seen that the contact stress is the maximal on these critical points by Eq. (1). It is difficult to calculate the contact stress on these locations. Usually, the pitch point is nearby, the difference of their contact stresses is very little. In the standard of ISO6336, the calculation of contact stress has been also taken the place of the pitch point \[4\]. This approximation is consistent with the fact that pitting mostly appears near the pitch point on the tooth profile.

Non-circular gears are the same as spur gears. It is also segmented into a single teeth-meshing area and a double teeth-meshing area on the tooth profile \[5\]. But the normal force is not equal in the single teeth-meshing area. So the point of the maximum contact stress is difficult to determine by Eq. (1) when the relative radius of curvature of two tooth profiles and the normal force are both variational. Based on Ref. [6], the normal force changes very little, usually less than 2% in the case of the transmission ratio \(i \leq 5\). In addition, the tooth profiles of non-circular gears and equivalent gears are also very similar. Their relative radii of curvature change consistently with each other based on Ref. [7]. Therefore, it is feasible that the contact stress is calculated by using equivalent gears instead of non-circular gears.

**4 Calculation of torque and rotation speed for equivalent gears**

There are many influence factors on the contact stress, but the normal force and sliding speed are the most important. To ensure the correct result, the power and rotation speed of the equivalent gears need to be calculated according to the normal force and sliding speed of the non-circular gears. Based on this condition,
the power and rotation speed of the equivalent gears are deduced when the normal force and rotation speed of the non-circular gears are known.

In Fig. 2, points o and o’ are rotation centers of the non-circular gears and the equivalent gears; $F_n$ and $F_n'$ are the normal forces; $l$ and $l'$ are their arms of force; $\alpha$ and $\alpha'$ are the pressure angles; $r(\varphi)$ and $r(\varphi)'$ are the gyration radiuses of pitch points; and $\varphi$ is the rotating angle of the non-circular gears. When the rotation speed of the non-circular gears is known, based on the equal of the tangential linear velocity of their pitch points, the rotation speed of the equivalent gears is

$$n' = \frac{r(\varphi)n\cos(\alpha'-\alpha)}{r(\varphi)'}.$$  

(2)

Fig. 2 The contrast of non-circular gears and equivalent gears

When the normal force of non-circular gears is known, based on the equal of their normal forces of the equivalent gear and the non-circular gear, the torque of the equivalent gear is

$$T' = \frac{F_n \times l'}{1000} = \frac{F_n \times l'}{1000}.$$  

(3)

Further, the transmitted power of the equivalent gear is

$$P' = \frac{F_n \times l' \times n'}{9.549 \times 10^5}.$$  

(4)

5 Finite element analysis of contact stress for non-circular gears

The FEM is an effective numerical method to solve the nonlinear contact-collision problem. To verify the above, the solving process of the FEM for the contact stress of non-circular gears is carried out. The detailed process could be referred to Ref. 2.

In the finite element analysis, the analysis model is usually divided into a single tooth model and a full teeth model. In consideration of the operation accuracy and teeth profile dissimilarity, the single tooth model is chosen as the solid model [8-10], as shown in Fig. 3a. Hexahedral element is used in the mesh dividing, which is suitable for complex curve and surface model. Contact pair is also build up in the teeth-meshing area. This treatment can effectively reduce the computation cost with high accuracy. In the FEM, boundary conditions should reflect the real constraint conditions as far as possible. For the gear engagement, the driven gear is pushed forward by the driving gear via the contact of teeth pairs [11-12]. So it is the prerequisite that the normal force of tooth profile is the same as that in the real situation. The torque should be loaded in line with the power and rotation rate of the non-circular gear pair. Fig. 3 is an example of the final analysis result for non-circular gears. Fig. 3a is the stress nephogram of a teeth pair on the tooth profile, and Fig. 3b is the one along the tooth width.

6 Comparative analysis of results of equivalent gears and FEM

The result of the equivalent gear is the one that is obtained by using equivalent gears instead of non-circular gears to calculate the contact stress. It usually requires the following steps. Firstly, equivalent gear
pair should be got according to different positions of the pitch circle. Their module is the same as non-circular gear, and radiiuses are also equal to the curvature radiiuses of the pitch curves. Secondly, transmission parameters of equivalent gear pair should be also converted. It must be ensured that the normal force and sliding speed of the equivalent gears are the same as those of the non-circular gears, and they can be got by calculating the power and rotation speed of equivalent gear based on Eqs. (2) to (4). Lastly, the contact stress can be calculated by the calculating standard of load capacity of cylinder gears. GB/T3480-1997 is used in the paper. The result is taken as the computational contact stress of the equivalent gear method.

The result of FEM is obtained by using the finite element technique to calculate the contact stress of non-circular gear. The solving process is the same as before. The maximum contact stress of tooth pair for non-circular gears could be obtained [13-14]. The result is taken as the computational contact stress of the FEM.

Fig. 4 shows fitting curves of the equivalent gear method and the FEM for the elliptic gear and oval gear. Through the comparison, variation trends of calculating results of two methods are the same, but the computational contact stresses of the equivalent gear method are about 0.08 times more than the analysis results of the FEM. This is mainly because that the calculating standard GB/T3480-1997 is prone to be safer, so its computational result is larger than the real value in application. The most important point is that the difference values are a little fewer at the limiting positions of the pitch circle, such as the 0°, 90°, 180°, 270° and 360° in Fig. 4b. It causes the curvature radius of pitch circle changes faster at these positions. Therefore, safety factor should be increased at these positions by using equivalent gears instead of non-circular gears to calculate the contact stress.

Fig. 3 Stress nephograms of the finite element method for non-circular gear of distribution a) on the tooth profile and b) along the axis of the gear

7 Conclusions

Based on the analysis about contact stress for non-circular gears, the following conclusions can be made:

1) In the transmission ratio \( i \leq 5 \), the tooth profile difference of non-circular gears and equivalent gears is very small. It can be ignored compared with the calculation error of Hertz formula itself. So it is
feasible and reliable by using equivalent gears instead of non-circular gears to calculate the contact stress.

2) The normal force, arm of force and relative radius of curvature for non-circular gears on the tooth profile are variational. There are many influence factors on the contact stress, but the normal force and sliding speed are the most important. To ensure the result correct, the normal force and sliding speed need to be in line with non-circular gears when the contact stress is calculated by equivalent gears.

3) Because of the quicker change of the radius at the limiting positions of the pitch circle, the computational contact stress inclines to be a small value by using equivalent gear instead of non-circular gears to calculate. Safety factors should be increased at these positions to get a more reliable result.

References


