Computer simulation of active suspension based on the full-vehicle model

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Abstract: The current method to solve the problem of active suspension control for a vehicle is often dealt with a quarter-car or half-car model. But it is not enough to use this kind of model for practical applications. In this paper, based on considering the influence of factors such as, seat and passengers, a MDOF(multi-degree-of-freedom) model describing the vehicle motion is set up. The MODF model, which is 8DOF of four independent suspensions and four wheel tracks, is more applicable by comparison of its analysis result with some conventional vehicle models. Therefore, it is more suitable to use the 8DOF full-car model than a conventional 4DOF half-car model in the active control design for car vibration. Based on the derived 8DOF model, a controller is designed by using LQ (linear quadratic) control theory, and the appropriate control scheme is selected by testing various performance indexes. Computer simulation is carried out for a passenger car running on a road with step disturbance and random road disturbance expressed by Power Spectral Density (PSD). Vibrations corresponding to ride comfort are derived under the foregoing road disturbances. The response results for uncontrolled and controlled system are compared. The response of vehicle vibration is greatly suppressed and quickly damped, which testifies the effect of the active suspension. The results achieved for various controllers are compared to investigate the influence of different control schemes on the control effect.
Key Word: active suspension; LQ control; 8DOF model

1. INTRODUCTION

Since the traditional suspension can hardly meet the demands for both ride comfort and handling stability when the performance of light weight and high speed of a vehicle are emphasized, more attention has been focused on the development of advanced active suspension and its control method has been researched widely.

The current method for solving the problem of active suspension control for a vehicle is often dealt with a quarter-car or half-car model [1,2]. But it is not enough to use this kind of model for practical applications. As is well known, the design of any control system depends crucially on the accuracy of the mathematical model used to describe the real system. A quarter-car or half-car model has shortcomings, for example, it is difficult to derive its control laws accounted for both of the two types of correlation between the inputs at the four wheels; i.e. the cross correlation between left and right hand tracks and the front-to-rear correlation arising from the fact that the rear wheel input is a delayed version of the front one. Based on considering the influence of factors such as seat, passengers, cross-correlation between left and right hand tracks, wheelbase time delay between the front and rear inputs, etc., a MDOF (multi-degree-of-freedom) model describing the vehicle motion is set up.

2. Vehicle Model

The full-vehicle model used in this work is shown in Fig. 1(a). The body and wheel masses are assumed to be rigid bodies, and eight degrees of freedom are used including body vertical, seat vertical, roll and pitch and vertical motion of each of the four wheels. The 2DOF and 5 DOF vehicle models are illustrated by Figs.1 (b) and (c) respectively.

2.1 Equations of motion

The equations of motion for the model in Fig.1(a) are given below.

$$\begin{aligned} m_1\ddot{y}_1 + (c_1+c_2)\dot{y}_1 - c_2\dot{y}_5 - (B/2)c_2\dot{y}_6 + L_1c_2\dot{y}_7 + \\ (k_1+k_2)y_1 - k_2y_5 - (B/2)k_2y_6 + L_1k_2y_7 + u_1 \\ = k_1y_{01} + c_1\dot{y}_{01} \end{aligned} \qquad (1)$$

$$\begin{aligned} m_2\ddot{y}_2 + (c_3+c_4)\dot{y}_2 - c_4\dot{y}_5 + (B/2)c_4\dot{y}_6 + L_1c_4\dot{y}_7 + \\ (k_3+k_4)y_2 - k_4y_5 + (B/2)k_4y_6 + L_1k_4y_7 + u_2 \\ = k_3y_{02} + c_3\dot{y}_{02} \end{aligned} \qquad (2)$$

$$\begin{aligned} m_3\ddot{y}_3 + (c_5+c_6)\dot{y}_3 - c_6\dot{y}_5 - (B/2)c_6\dot{y}_6 - L_2c_6\dot{y}_7 + \\ (k_5+k_6)y_3 - k_6y_5 - (B/2)k_6y_6 - L_2k_6y_7 + u_3 \\ = k_5y_{03} + c_5\dot{y}_{03} \end{aligned} \qquad (3)$$

$$\begin{aligned} m_4\ddot{y}_4 + (c_7+c_8)\dot{y}_4 - c_8\dot{y}_5 + (B/2)c_8\dot{y}_6 - L_2c_8\dot{y}_7 + \\ (k_7+k_8)y_4 - k_8y_5 + (B/2)k_8y_6 - L_2k_6y_7 + u_4 \\ = k_7y_{04} + c_7\dot{y}_{04} \end{aligned} \qquad (4)$$

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$$\begin{aligned} & m_5\ddot{y}_5 - c_2\dot{y}_1 - c_4\dot{y}_2 - c_6\dot{y}_3 - c_8\dot{y}_4 + (c_2+c_4+c_6+c_8+c_9)\dot{y}_5 + [(B/2)(c_2-c_4+c_6-c_8)+L_3c_9]\dot{y}_6 \\ & -[L_1(c_2+c_4)-L_2(c_6+c_8)-L_4c_9]\dot{y}_7 - c_9\dot{y}_8 - k_2y_1 - k_4y_2 - k_6y_3 - k_8y_4 + (k_2+k_4+k_6 \\ & +k_8)y_5 + [(B/2)(k_2-k_4+k_6-k_8)+L_3k_9]y_6 - [L_1(k_2+k_4)-L_2(k_6+k_8)-L_4k_9]y_7 - k_9y_8 \\ & -u_1-u_2-u_3-u_4=0 \end{aligned} \tag{5}$$

$$\begin{aligned} & m_6\ddot{y}_6 - (B/2)c_2\dot{y}_1 + (B/2)c_4\dot{y}_2 - (B/2)c_6\dot{y}_3 + (B/2)c_8\dot{y}_4 + [(B/2)\left(c_2-c_4+c_6-c_8\right)+L_3c_9]\dot{y}_5 \\ & +[(B/2)^2\left(c_2+c_4+c_6+c_8\right)+L_3^2c_9]\dot{y}_6 - [(B/2)L_1\left(c_2-c_4\right)-(B/2)L_2\left(c_6-c_8\right)-L_3L_4c_9]\dot{y}_7 \\ & -L_3c_9\dot{y}_8 - (B/2)k_2y_1 + (B/2)k_4y_2 - (B/2)k_6y_3 + (B/2)k_8y_4 + [(B/2)\left(k_2-k_4+k_6-k_8\right) \\ & +L_3k_9]y_5 + [(B/2)^2(k_2+k_4+k_6+k_8)+L_3^2k_9]y_6 - [(B/2)L_1\left(k_2-k_4\right)-(B/2)L_2\left(k_6-k_8\right) \\ & -L_3L_4k_9]y_7 - L_3k_9y_8 + (B/2)(u_1-u_2+u_3-u_4)=0 \end{aligned} \tag{6}$$

$$\begin{aligned} & m_7\ddot{y}_7 + L_1c_2\dot{y}_1 + L_1c_4\dot{y}_2 - L_2c_6\dot{y}_3 - L_2c_8\dot{y}_4 - L_1(c_2+c_4) - L_2(c_6+c_8) - L_4c_9]\dot{y}_5 - [(B/2)L_1(c_2-c_4) \\ & -(B/2)L_2(c_6-c_8) - L_3L_4c_9]\dot{y}_6 + [L_1^2(c_2+c_4)+L_2^2(c_6+c_8)+L_4^2c_9]\dot{y}_7 - L_4c_9\dot{y}_8 + L_1k_2y_1 + L_2k_4y_2 \\ & -L_2k_6y_3 - L_2k_8y_4 - [L_1(k_2+k_4)-L_2(k_6+k_8)-L_4k_9]y_5 + [(B/2)L_1(k_2-k_4)-(B/2)L_2(k_6-k_8) \\ & +L_3L_4k_9]y_6 + [L_1^2(k_2+k_4)+L_2^2(k_6+k_8)+L_4^2k_9]y_7 - L_4k_9y_8 - L_1u_1 - L_1u_2 + L_2u_3 + L_2u_4 = 0 \end{aligned} \tag{7}$$

$$m_8\ddot{y}_8 - c_9\dot{y}_5 - L_3c_9\dot{y}_6 - L_4c_9\dot{y}_7 + c_9\dot{y}_8 - k_9y_5 - L_3k_{96} - L_4k_9y_7 + k_9y_8 = 0 \tag{8}$$

where y_1, y_2, y_3, and y_4 are the vertical distance of unsprung weight of left front, left rear, right front and right rear, respectively; y_5, y_6 and y_7 are respectively the vertical distance, the roll angular displacement and the pitch angular displacement of the center of the body mass; y_8 is the vertical distance of the man-seat system; B is the tread; L is wheelbase; L_1 and L_2 are the distances respectively from the back axle and the front axle to the center of mass; L_3 and L_4 are the distances between the joint of the man-seat system and the body center of mass respectively in the body-width direction and the longitudinal direction; m_1, m_2, m_3 and m_4 are the unsprung weight of left front, left rear, right front and right rear, respectively; m_5 is the body mass; m_6 is the roll moment of inertia; m_7 is the pitch moment of inertia; m_8 is the weight of man-seat system; k_1, k_2,..., k_9 are respectively the elastic coefficients of the corresponding springs shown in Fig. 1; and c_1, c_2,...,c_9 are respectively the damping factors of according dampers.

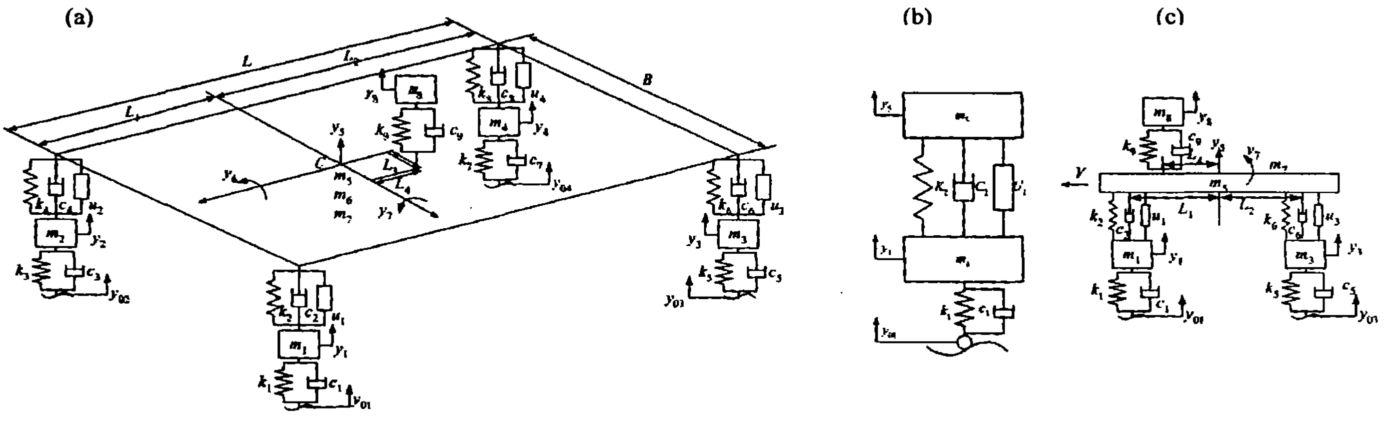

Fig. 1. The vehicle models: (a) 8DOF, (b) 2DOF, and (c) 5DOF

2.2 State space equation

The state space equation is given by

$$\dot{x} = Ax + Bu + G\omega \tag{9}$$

where A, B and G are three matrixes; ω is the system noise; and x is the state variable vector defined as

$$\begin{aligned} x = \{ & y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, \\ & \dot{y}_1, \dot{y}_2, \dot{y}_3, \dot{y}_4, \dot{y}_5, \dot{y}_6, \dot{y}_7, \dot{y}_8\}^T \end{aligned} \tag{10}$$

The equations of motion and state space equation for 2DOF and 5DOF vehicle model can also be described in the same way as above.

2.3 Comparison of vehicle models

Considering the influence caused by the seat, the cross-correlation between left and right track inputs, and the wheelbase time delay between the front and rear inputs to the vehicle model, the frequency responses of various passive vehicle model for the same road inputs are calculated.

By comparing the results shown in Fig.2, it can be seen that:

a) The seat and passengers strongly affect the vibration of the car, which is related to the ride comfort.

b) The influence of the cross-correlation between left and right track inputs and wheelbase time delay between the front and rear inputs is not neglectable.

It is not enough only to consider the factors of the seat, the passengers, the cross-correlation between left and right track inputs. The wheelbase time delay between front and rear inputs in general should also be considered, for their influence on vibration can not be ignored. So, vibration control is indispensable.

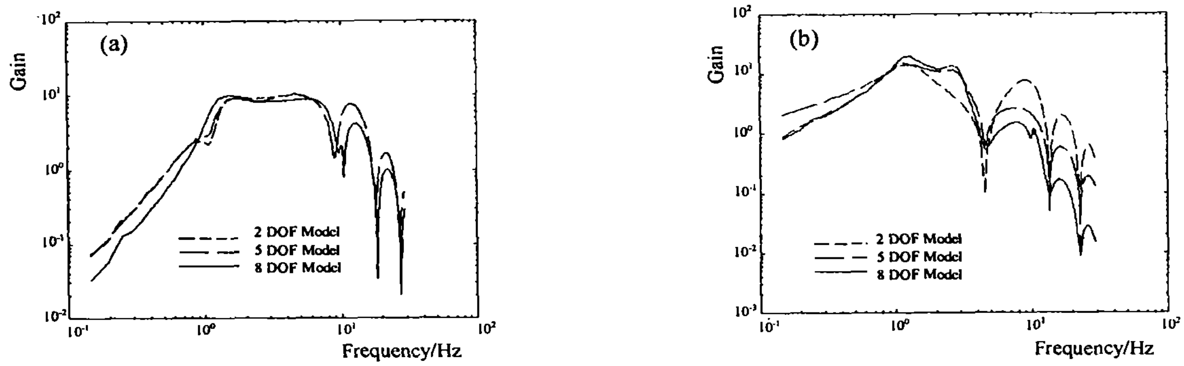

Fig. 2. Frequency response of vehicle vibration under random road input: (a) vehicle body pitch acceleration; and (b) seat acceleration.

3. The Controller Design

In order to both avoid the complexity and improve the precision in designing, a controller using the simple model in Fig.3 is designed. In this model, the vertical motion of the seat is neglected.

When the active suspension controller is designed, the input of a control system is assumed as $\boldsymbol{u}= -k\boldsymbol{x}$, where the feedback gain $\boldsymbol{k}$ is determined by using LQ optimal control theory.

According to Fig. 2, the state space equation is given by Eq. (10) and $\boldsymbol{u}(t)= -k\boldsymbol{x}(t)$

The cost function for this system is expressed as:

$$J = \int_0^{\infty} [y^{\mathrm{T}} \boldsymbol{Q} y + \boldsymbol{u}^{\mathrm{T}} \boldsymbol{R} \boldsymbol{u}] \mathrm{d}t$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the matrices of weighting coefficients. As is well known, the feedback gain vector $\boldsymbol{k}$ of the control system can be computed by solving a Riccati equation. So the optimal control input $\boldsymbol{u}$ is obtained and is expressed as $\boldsymbol{u}(t)= -k\boldsymbol{x}(t)$.

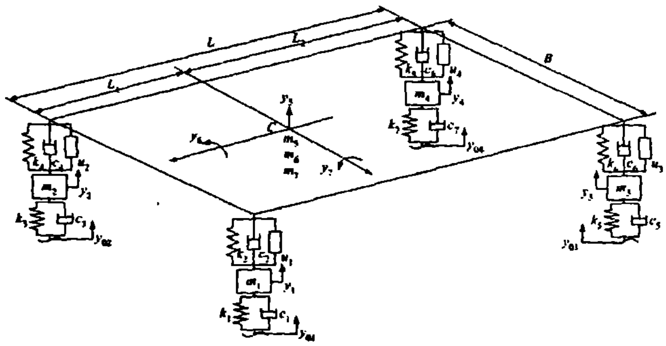

Fig.3. The 7DOF vehicle model

4. Results and Discussions

Computer simulation is carried out for a passenger car running on a road with step disturbance and random road disturbance expressed by Power Spectral Density (PSD). The vibrations corresponding to ride comfort are derived under the foregoing road disturbances.

4.1. Frequency response under random road disturbance

Fig.4 illustrates the responses of the vehicle at a velocity of 72 km/h under random road disturbance. Figs.4(a) and (b) show the responses of the car body and seat acceleration power spectral density under this disturbance, respectively.

The results of the responses of the uncontrolled and controlled systems are compared. The response of vehicle vibration is greatly suppressed and quickly damped, which verifies the effect of the active suspension.

4.2 Response under step road disturbance

Fig.5 illustrates the response of the vehicle at a velocity of 72 km/h under the step road disturbance. The vibrations are greatly suppressed by the active control over most of the frequency range of which the vibration strongly affects ride comfort.

From the results mentioned above, it can be found that:

a) through applying the controllers designed by the 7DOF model to the 8DOF vehicle model, it is demonstrated that the seat, passengers, cross-correlation between left and right hand tracks, and wheelbase time delay between front and rear inputs greatly influence the control effect.

b) The controller designed by using the 7DOF model reduces not only the vibration of the seat, but also the suspension working space. For both high precision and low cost, it is feasible to design the controller based on the 7DOF model.

5. CONCLUSIONS

Based on the calculated results of the full-car model with active suspension controlled according to LQ theory, which have been described previously, the following conclusions can be drawn.

a) The results show that the acceleration of the body and seat are reduced obviously. So the 8DOF vehicle model is applicable.

b) The inclusion of the effect of cross-correlation between the left and right track profiles in the derivation of the control laws does not result in any improvements in performance. Hence, it can be ignored in deducing control algorithms for the full vehicle model.

c) The inclusion of the effect of the front to rear wheel correlation due to the wheel base time delay is, however, very important to the deduction of the control algorithms.

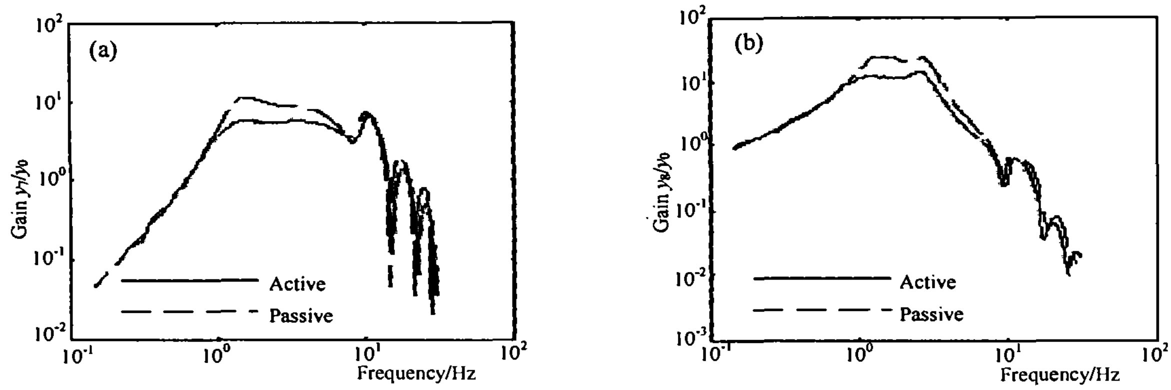

Fig.4. Frequency responses of active and passive system under random road inputs: (a) body pitch acceleration, and (b) seat acceleration

Legend: Active ——— Passive - - - - -

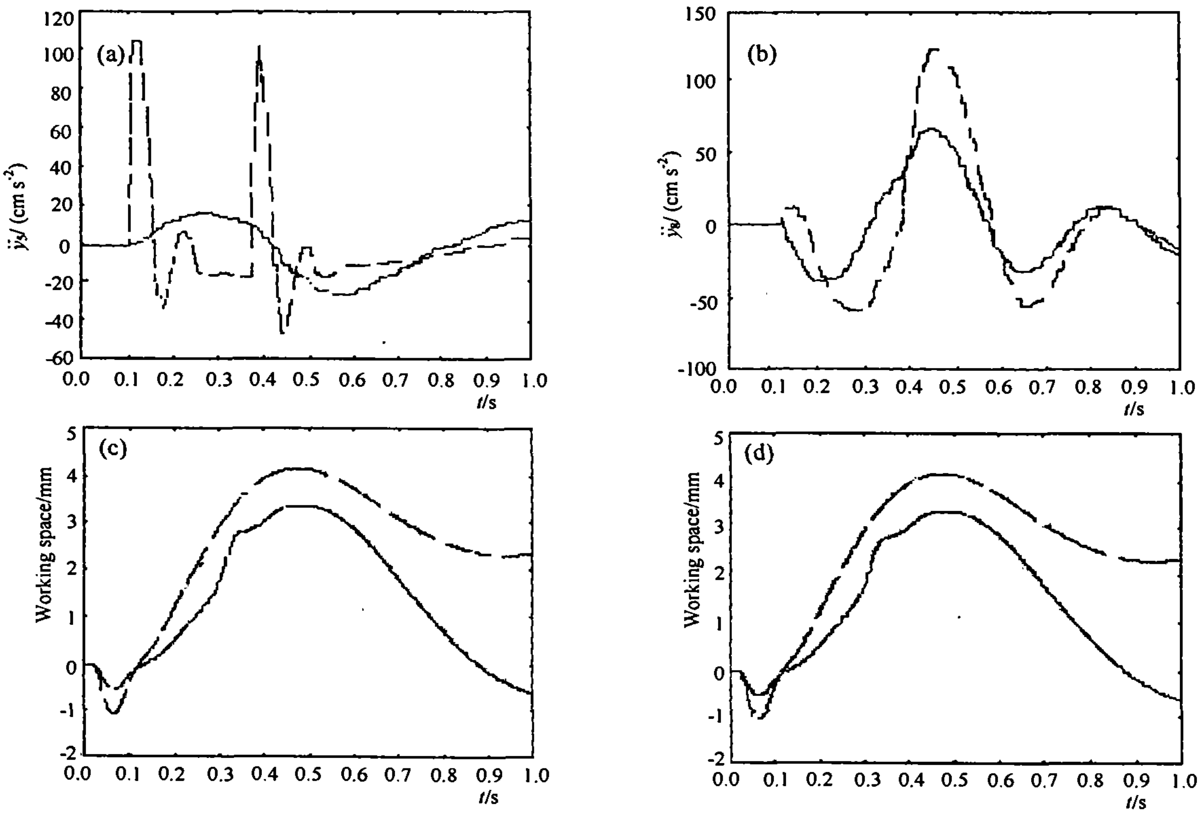

Fig.5. Time history of active and passive systems under step road inputs: (a) body acceleration, (b) seat acceleration, (c) left (front) suspension working space, and (d) right(front)suspension working space.

d) By combining the 8DOF vehicle model and 7DOF control model, the control results are greatly improved.

e) As seat, passengers, and wheelbase time delay between front and rear inputs have a great influence on the performance of the control system, these factors should be considered in the design of the controller.

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