Application of dynamic programming to structural repairing strategies*

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Abstract: A model of dynamic programming for repairing strategies of concrete structures during a projected service period is proposed, which takes into account the degradation in strength of components and the probability of accidental load. This model takes the safety grade of a structural system as the state variable of repairing strategies, and incorporates economic factors including expected repair cost, property loss due to structure failure, goods and material loss due to structure failure, loss of production interrupt due to structure failure, and inspection cost in decision making. It is found that the optimal repairing strategies are sensitive to the probability of accidental loads as well as the failure costs. The practicality of the model is demonstrated by an example.
Keywords: structural reliability; safety classification; repairing strategies

1. Introduction

Within the last decade, the recent achievements in modern reliability theory, decision theory and optimization theory have been used for the planning optimization of design, maintenance and repairing strategies for structures. Consideration of the time-variant reliability of structures which is an important aspect of structural reliability analysis has drawn much attention [1,2]. It is well known that environmental stresses such as carbonation of concrete and corrosion of reinforcement always attack structural integrity gradually. The accidental loads such as great earthquakes and fire often result in damage, even destruction to structures. An ideal solution to reduce failure costs of structures may resort to more accurate prediction of accidental event, the structural degradation model, and hazard reduction preparedness [3,4].

In this paper, a dynamic programming model for repairing strategies to minimize the total expected cost of structures and maintain their reliability at or over an established target level during its projected service life is proposed. The states of the model are defined as the safety grades of a structural system with each grade corresponding to a given region of structural system reliability and a specific repair effort.

2. Structural Reliability

Great progress has been reported in the theory of structural reliability in the past two decades. One milestone is marked by the important developments in this field, due to the well-known interest of engineers and researchers in system behaviors instead of only component failure events [5]. So the optimum design and repairing strategies are based on the reliability of a structural system instead of that of components [6]. As it is well known, in its expected lifetime, a structural system may experience various types of deterioration or damage, such as corrosion, fatigue, freeze-thaw and accidental losses of structural material and capacity due to great earthquakes, strenuous vibration, and man made errors in design and construction. The deterioration and damage may result in a substantial decrease of structures. In comparison with the structural strength, the uncertainties and variability of accidental load are much more considerable. They are dominant factors for reliability-based design and repair decision. Therefore, the analysis of structural reliability during the lifetime of a structural system should consider both the time-variant strength of structures and time-variant load, especially the accidental load such as great earthquakes.

Since an accidental load occurs randomly, it can be regarded as a discrete random variable. Let I be the random affair of an accidental load, $P_I(t)$ be the occurrence probability of an accidental load I at time t, $P_{f_I}(t)$ be the failure probability of a structure under the action of the accidental load I, while t is the time variable. It is known by the theory of conditional probability that $P_f(t)$, the structural failure probability at time t, is related to $P_I(t)$ and $P_{f_I}(t)$, and is evaluated by

$$P_f(t)=\sum_{I=I_1,I_2,I_3} P_{f_I}(t)P_I(t) \quad (1)$$

where, I_1, I_2 and I_3 are three different affairs of accidental loads.

Assuming that the strength is independent of the load, $P_{f_I}(t)$ is determined by

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$$P_{f_t}(t)=P\{R(t)-S(t)<0\}=\int_0^{\infty} f_s(s,t)F_R(s,t)\mathrm{d}s =\int_0^{\infty} f_R(r,t)[1-F_s(r,t)]\mathrm{d}r \tag{2}$$

where $R(t)$ is the time-variant strength of structure, and $f_R(r, t)$ the density distribution function of $R(t)$; $S(t)$ the time-variant load effect of structure; $f_S(s,t)$ the density distribution function of $S(t)$; and $F_R(s,t)$ and $F_S(r,t)$ are respectively the lateral accumulation distribution function of $R(t)$ and $S(t)$.

Considering the degradation in the strength of a structures, the structural capacity $R(t)$ generally is

$$R(t)= R_0\, g(t) \tag{3}$$

where R_0 is the original component capacity; and $g(t)$ is the time-dependent degradation function defining the fraction of strength with respect to R_0. The form of the degradation function is dependent on the rule of deterioration. For example, considering the corrosion of steel in concrete, the change in the structural capacity is insignificant during the initiation period, so $g(t)$ can be assumed as

$$g(t)=\begin{cases}1, & t<t_i\\ 1-c(t-t_i), & t\ge t_i\end{cases} \tag{4}$$

where c is the degradation rate of the component capacity (Mori, 1994).

Assume that a structural system is considered to fail if it becomes a mechanism. The so called "Branch-and-Bound" method, as originally proposed by Murotsu et al [7], which is applicable to truss and frame structures, can be used to calculate the failure probability of the structural system. As this method is based on the idealization of plastic hinge formation, the sequence of plastic hinges is thus determined by the probability of a cross section satisfying the yield condition.

3. Structural Safety Classification

In practice, it is quite usual to use the grade of safety and reliability of structural components or structural system instead of reliability index or failure probability in order to indicate the state of structural safety and reliability distinctly. For example, there are 4 grades of appraisal criteria for structural safety in the Reliability Appraisal Criteria for Industrial Buildings of China (RACIBC, GBJ144-90, 1989) [8], in which a component reliability satisfying the current design code and needing no repair effort is rated in the structural component safety grade a, that slightly lower than the design code and needing no repair effort falls into grade b, that failing to meet the design code and in need of repair is classified as grade c, and that far below the design code and in need of immediate repair is rated in grade d. However, the structural system safety grade A indicates that the reliabilities of dominant components of a structure satisfy the current design code while the reliabilities of less important components are lightly lower than the design code, and the structural system needs no repair effort; grade B refers to that the reliabilities of dominant components are lightly lower than the design code while the reliabilities of less important components don't satisfy the current design code, and the structural system need be retrofitted; grade C means that the reliabilities of dominant components do not satisfy the design code and the system needs repairing; and grade D signifies that the reliabilities of dominant components are far below the design code and the system needs immediate repairing. Almost all the criteria mentioned above are established according to the experiential knowledge in engineering. Although they are simple and distinct in practice, they are inaccurate because of the weakness of computation compared with the classic reliability theory. It is helpful to work out the relationship between the failure probability and the safety grade of structures [9].

According to the current design codes in China, the critical reliability indexes of a structural component to each safety grade in RACIBC are β_{a_0} =3.2 for grade a, β_{b_0} =2.7 for grade b, and β_{c_0} =2.3 for grade c. The safety classification of a structural component is illustrated in Table 1.

Table 1 Safety classification of structural components

Grade a	Grade b	Grade c	Grade d
$\beta\ge\beta_{a_0}$	$\beta_{b_0}\le\beta\le\beta_{a_0}$	$\beta_{c_0}\le\beta\le\beta_{b_0}$	$\beta<\beta_{c_0}$

Considering a virtual structural system with all components being in the same critical state grade a and having the same reliability index β_{a_0} =3.2, an ideal critical structural system of grade A can be defined as

$$I=\{\beta_i\,|\beta_i=\beta_{a_0};\ i=1, 2, \ldots, n\}=I_A \tag{5}$$

where I is the structural system; β_i is the reliability index of the structural component i; i=1, 2, …,n; n is the total number of components in the appraised structure; I_A is the ideal critical structural system of grade A. The ideal critical structural system of grades B and C can also be respectively described as

$$I=\{\beta_i\,|\beta_i=\beta_{b_0};\ i=1, 2, \ldots, n\}=I_B \tag{6}$$

$$I=\{\beta_i\,|\beta_i=\beta_{c_0};\ i=1, 2, \ldots, n\}=I_C \tag{7}$$

The reliability or failure probability of an ideal critical structural system can be evaluated by modern reliability analysis method of structural system such as "Branch-and-Bound" method. An appraised structure with a failure probability P_f is regarded as a structural system of grade A if P_f is not grater than P_{f_A} ,the failure probability of the ideal critical structural system of grade A. It is regarded as grade B if $P_{f_A}<P_f\le P_{f_B}$. The safety classification of a structural system is indicated in Table 2.

Table 2 Safety classification of structural systems

class A	class B	class C	class D
$P_f\le P_{f_A}$	$P_{f_A}<P_f\le P_{f_B}$	$P_{f_B}<P_f\le P_{f_C}$	$P_f>P_{f_C}$

4. Dynamic Programming for Repairing Strategies

4.1 Expected cost of repair decision

Let the safety grades of a structural system be the state variables of repairing strategies and take the following economical factors in the decision model.

4.1.1 Expected repair cost of structures C_r

The expected repair cost of structures C_r is given by

$$C_r(t)=\alpha P_f(t)\Delta R(t)(1+r)^{-t}=\alpha\, P_f(t)(P_f(t)-P_f^0)(1+r)^{-t} \quad (8)$$

where P_f^0 is the failure probability of un-repaired structure; $P_f(t)$ the time-variant failure probability of repaired structure; α the constant of repair cost; and r the real interest rate.

4.1.2 Property loss due to structural failure D_s

The property loss due to structural failure Ds is given by

$$D_s = D_s^{\,i}\, C_0(1+r)^{-t}, \qquad i=\text{A, B, C and D} \quad (9)$$

where D_s^A, D_s^B, D_s^C, and D_s^D are the constants of property losses respectively related to structural safety grades A, B, C and D; and C_0 is the initial construction cost of structure.

4.1.3 Goods and materials loss due to structural failure D_m

A structural failure often causes goods and materials loss D_m of structures, which may exceed D_s and even the property value of the original structure. It is given by

$$D_m = D_m^{\,i}\, C_1(1+r)^{-t}, \qquad i=\text{A, B, C and D} \quad (10)$$

where D_m^A, D_m^B, D_m^C, and D_m^D are the constant parameters of goods and materials loss of a structure; C_1 the original value of goods and materials of the structure.

4.1.4 Loss of production interrupt due to structural failure D_p

The loss of production interrupt D_p due to a structural failure is evaluated by

$$D_p = Pt_p \quad (11)$$

where P is the output value per unit time; and t_p the whole time of production interrupt.

4.1.5 Total expected failure cost of structures C_f

The total expected failure cost of structures (TEFCS) C_f is calculated by

$$C_f(t)=\int_{t_1}^{t_2}(D_s+D_m+D_p)P_f(t)\mathrm{d}t \quad (12)$$

4.1.6 Inspection cost C_i

The inspection cost C_i is given by

$$C_i= C_i(S)C_0 \quad (13)$$

where $C_i(S)$ is the constant relating to a structural form.

4.2 Repairing Strategies

The repairing strategies are developed by solving the following optimization problem in which the total expected cost (including inspection cost C_i, repair cost C_r and total expected failure cost C_f) in the projected lifetime of structure are minimized subjected to the allowable failure probability, the allowable repair cost and the failure cost.

$$\min C=C_r + C_f + C_i \quad (14)$$

$$\text{s.t. } P_f(t)\le [P_f] \quad \text{(a)}$$

$$C_r(t)\le [C_r] \quad \text{(b)}$$

$$C_f(t)\le [C_f] \quad \text{(c)}$$

where, $[C_r]$ is the allowable repair cost; $[C_f]$ is the allowable total expected failure cost; $[P_f]$ is the allowable structural failure probability.

The optimal repairing strategies are based on the failure probability and the safety grade of a structural system. It is assumed that n times of repairs are performed in several stages at time T_1, T_2, ... ,T_n with repair efforts R_1, R_2, ... , R_n (which may be inspection, rehabilitation, repairing, retrofit or demolishment). In each stage, it is needed to minimize the partial expected cost during $[0, T_i]$(i=1, 2, ..., n) so as to make the total expected cost during the lifetime minimum. The interval $[T_i, T_{i-1}]$ of the decision stage may be different considering the character of the time-variant reliability of a structure during its lifetime. For example, it may be shorter during the earlier time of the structural lifetime because of the lower failure probability and the lower risk of the structure in this stage, while a longer interval should be employed for an aged period because of the higher failure probability and higher risk of structure. The decision process can be modeled with a dynamic programming as shown in Fig. 1.

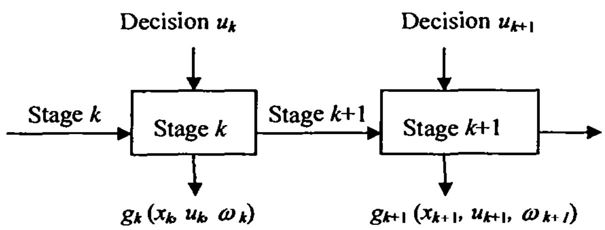

Fig.1. The procedure of dynamic programming of repairing strategies

It can be expressed as

$$C_k(P_f(t_k))=\min\{w_k(P_f(t_k), x_k)+ C_{k+1}(P_f(t_{k+1}))\} \quad (15)$$

where $P_f(t_k)$ is the failure probability of structural system in stage k; x_k is the state variable in stage k (k=1, 2, ..., N, and N is the total number of decision stages), which is grade A, B, C, or D; $w_k(P_f(t_k), x_k)$ is the partial loss in the decision stage k; $C_k(P_f(t_k))$ is the partial expected cost from the beginning to stage k.

5. Example

To explicate the application of the model, a two-span and two-story frame structure shown in Fig.2 is considered. It is assumed that the time-variant degradation function is $g(t)$=1−0.007(t−t_p) [4] while t_p=5 a. The time-variant occurrence probability of accidental load is shown in Fig.3, in which the probabilities of at least one earthquake with its intensity I greater than 6.5 occurs in 50 years are given by Poison process model [2,10]. The projected lifetime T is 50 a which is divided into 8 stages as (0 to 10) a, (10 to 20) a, (20 to 25) a, (25 to 30) a, (30 to 35) a, (35 to 40) a, (40 to 45) a, and (45 to 50) a. The initial construction cost of structure C_0 is 1 million Chinese yuan, and the goods and material cost in the structure C_1 is $2C_0$. Let the safety grades of the structural system be the state variables of repairing strategies, while the successive states occupancies in the decision model are governed by the accidental load occurrence probability, the effect of degradation of materials, and repair efforts. The transition relationships between repair efforts and safety grades are shown in Table 3. The structural repairing strategies are obtained by using the dynamic programming described by Eqs.(14) and (15).

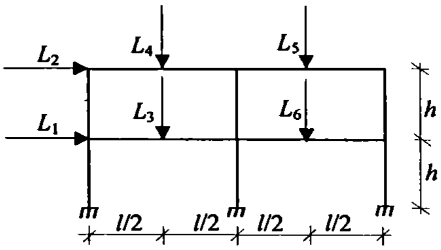

Fig. 2. Two-Span and Two-Story Frame Structure

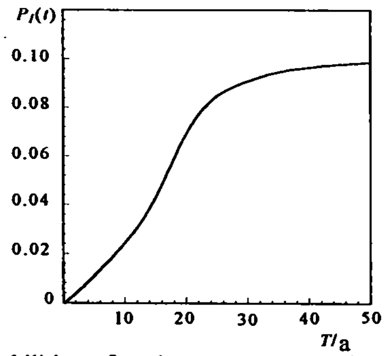

Fig.3. Probabilities of at least one event with its intensity I greater than 6.5 in next 50 years given an earthquake has just occurred(t_0=0 a) with Poison process model

The effect of accidental load occurrence model on the total expected failure cost of structure C_f with different C_0, C_1, and P, and the effect on the repairing strategies are indicated in Table 4. It can be seen that C_f increases with C_0, C_1 and D_p. The smaller the interest rate, the smaller C_f is and the smaller the effect of different earthquake process model on the evaluation of C_f and structural repairing strategies.

Table 3 Transition of Safety grades by required repair efforts

Repair effort	Safety grade			
	A	B	C	D
Inspection	A	B	C	D
Rehabilitation		A	B	C
Repairing			A	B
Retrofit or demolish				A

Table 4. Effect of seismic process model on repairing strategies

$C_0+C_1+D_p$ /(10^6 yuan)	Repairing strategies& change of safety grade	C_f/(10^6 yuan)	
		Before repair	After repair
300	35 a to 40 a, C→B	41.57	32.52
	35 a to 40 a, C→B	45.32	35.17
	30 a to 35 a, C→B	47.76	40.15
3000	25 a to 30 a,C→B; 45 a to 50 a,C→B	523.54	487.98
	25 a to 30 a,C→B; 40 a to 45 a,C→B	586.64	526.47
	21 a to 25 a,C→B; 35 a to 40 a,C→B	741.54	598.07
10000	25 a to 30 a,C→B; 45 a to 50 a,C→B	2262.4	1876.4
	25 a to 30 a,C→B; 40 a to 45 a,C→B	2484.5	1975.5
	21 a to 25 a,C→B; 35 a to 40 a,C→B	2854.5	2257.5

6. Conclusion

The repairing strategies of structures are dependent on not only the structural reliability or failure probability, but also the total expected cost of structures during their lifetime, people's allowability of failure risk, political reasons, and so on. Many of these factors are indeterminate and difficult to be evaluated. A reasonable decision should resort to the combination of qualitative analysis such as practical evaluation criteria, with quantitative analysis such as structural system reliability analysis and optimum theories.

In this decision model, the occurrence of an accidental load is described as a discrete random event, the time-variant structural reliability is denoted as four distinct safety grade with each grade corresponding to a special repair effort, and the given decision period is also split into several stages with a different interval considering the time-variant characteristic of the structural reliability during its service time, so as to make the dynamic programming model easy to be used, to minimize the total expected cost, and to maintain the structural failure probability at an established target.

It is shown that the evaluation of the total expected failure cost of structures and repairing strategies is sensitive to the occurrence probability of loads especially, and also on the failure cost of structures.

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