Backlash analysis of a new planetary gearing with internal gear ring

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Abstract: Due to low carrying capacity and short life of current ring reducer, we proposed a new planetary gearing with internal gear ring, which consisted of two stages. One was an involute planetary drive and the other was an N-type planetary drive with small teeth difference. The kinematic calculation was conducted based on the analysis of structural composition and working principle. Formulas of backlash calculation for the new planetary gearing were derived by using probabilistic theory and the analytical model was created at the same time. Then, main factors that affect the systematic backlash were introduced and the effects of manufacture error, misalignments and roller bearing parameters on the distribution of backlash were presented. The influence of gear backlash on systematic return difference was performed based on the backlash mathematic model proposed. The results show that the backlash is a little large, of which the backlash of bearing takes a greater part than two gearing stages. So we presented some practical methods to reduce the systematic backlash.

Keywords: planetary gearing with internal gear ring; probabilistic theory; return difference; gear backlash

1 Introduction

The planetary gearing is widely used in aerospace and industrial machinery due to its large transmission ratio, high carrying capacity and impact structure. Some new and higher requirements for mechanical transmission, such as larger transmission ratio, lighter weight, higher precision and lower backlash, are presented in various industrial fields, especially for high speed planetary drive systems [1-2]. With the rapid development of aerospace and science technology, higher demands are also asked to the country’s manufacture industry, including accuracy of manufacture, reliability, strength and material, among which, the systematic accuracy is obviously crucial to the general level of manufacture industry [3-5]. Therefore, it is meaningful to conduct the research of improving the systematic precision of planetary drive. Researchers have done a lot of work on the transmission accuracy analysis of planetary drive. Blanche and Yang [6-7] discussed the influence of manufacture error of single cycloid gear on rotation accuracy, while HIDAKAT from Yamaguchi University studied the rotation accuracy of two-stage cycloid drive structure and conducted the test. Wu, et al. [8-9] investigated the return difference of RV drive. In this paper, the probabilistic method [10-13] was used to perform the systematic backlash analysis of a new planetary drive with internal gear ring. Some practical methods were put forward to reduce the backlash which could be beneficial to the design of planetary drive.
2 Structural form and transmission principle

2.1 Basic structures

Fig. 1 shows the structural composition of the new planetary gearing with internal gear ring. It consists of two stages. The first stage is involute planetary drive transmission and the second is the N-type planetary drive with small teeth difference. The planet gear \( b \) is jointed with arm \( h \) to transmit power to the second stage. The power is inputted from shaft \( A \) and outputted from shaft \( B \). When the ring gear \( a \) rotates clockwise, the planet gear \( b \) will rotate around its own axis while rotate around the mechanism’s axis. Then the external gear \( 1 \) of second stage will do the plane motion driven by arm \( h \). The external gear will also lead the output mechanism \( H \) to rotate clockwise due to the fixed internal gear ring \( 2 \). The two stages composed of one outer gearing K-V gear train and two parallel inner gearing K-V gear trains [14] are coupled into a hermetic drive.

Fig. 2 shows the internal structure and Table 1 lists parameters of prototype which is designed as the structure above. Rated power of the prototype is 0.55 kw and input shaft speed is 1 500 r/min.

![Fig. 1 Schematic of kinematics](image1)

![Fig. 2 Structure chart of the new planetary drive](image2)

2.2 Kinematics analysis

As shown in Fig. 1, when the ring gear rotates clockwise, the planet gear \( b \) will rotate around its own axis anticlockwise while rotate around the mechanism’s axis. Then the external gear \( 1 \) of second stage will do the plane motion driven by arm \( h \). The external gear \( 1 \) will also lead the output mechanism \( H \) to rotate clockwise due to the fixed internal gear ring \( 2 \). The internal structure can be clearly seen in Fig. 3 which is the breakdown drawing of prototype. Then the output motion can be obtained through output mechanism \( H \) which is clockwise. So the transmission ratio is

\[
i = 1 + \frac{z_a}{z_2} \times \frac{z_2}{z_2 - z_1},
\]

where \( z_a \) is the tooth number of ring gear for the 1st stage, \( z_2 \) is the tooth number of planet gear for the 1st stage, \( z_1 \) is the tooth number of external gear for the 2nd stage, and \( z_2 \) is the tooth number of internal gear ring for the 2nd stage.

![Table 1 Prototype parameters](image3)

<table>
<thead>
<tr>
<th>Modulus/mm</th>
<th>Tooth number</th>
<th>Modification coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive gear</td>
<td>Drive gear</td>
<td>Drive gear</td>
</tr>
<tr>
<td>1st stage</td>
<td>0.8</td>
<td>25</td>
</tr>
<tr>
<td>2nd stage</td>
<td>2.0</td>
<td>61</td>
</tr>
</tbody>
</table>

Substituting the geometry parameters into Eq. (1), the transmission ratio can be represented by
3 Mathematical model of backlash

The new planetary gearing is a less differential gear drive which consists of a general involute planetary gear drive and a planetary drive with small teeth difference. The systematic backlash is composed of three parts which are from the first stage, second stage and the bearing system. The main errors and their regularities of distribution which affects the backlash should be determined, and then a probabilistic method is introduced to compute the backlash. The backlash calculated above should be converted to the output shaft, and the systematic backlash can be represented by [15]

\[
\Delta \phi \approx \Delta \phi_{g1} + \Delta \phi_{g2} + \Delta \phi_u, \\
T_{\Delta e} = \sqrt{T_{\Delta e,1}^2 + T_{\Delta e,2}^2 + T_{\Delta u}^2},
\]

(2)

where \( \Delta \phi_{g1} \) and \( T_{\Delta e,1} \) are the mean value and tolerance, \( \Delta \phi_{g2} \) and \( T_{\Delta e,2} \) are the mean value and tolerance of geometric backlash of involute planetary drive in the first stage, \( \Delta \phi_u \) and \( T_{\Delta u} \) are the mean value and tolerance of geometric backlash of involute planetary drive with small teeth difference in the second stage, and \( \Delta \phi_u \) and \( T_{\Delta u} \) are the mean value and tolerance of geometric backlash of bearing.

The total backlash caused by various factors of the new planetary gearing system is

\[
\Delta \phi \approx \Delta \phi_{g1} \pm \frac{T_{\Delta e}}{2}.
\]

(3)

3.1 Backlash analysis for the first stage

For the first stage, main factors that affect the backlash are: a) manufacture errors including reduction of tooth thickness and gear eccentricity which could be represented by gear radial integrated error and average length deviation of common normal line; b) machine tool errors and assembly errors of shaft, bearing and gearbox, where, we only take the offset error into account; and c) other errors which are caused by temperature and elastic deformation. These errors are ignored since they have very little influence on the systematic backlash.

3.1.1 Average deviation of common normal line

Fig. 4 shows that \( W \) is the length of normal line, and \( \alpha \) is the angle between \( W \) and \( AB \) which is equal to the pressure angle. \( E_{\Delta m} \), \( E_{\Delta w} \) and \( E_{\Delta u} \) are respectively the upper deviation, lower deviation, and tolerance, among which, \( \Delta E_{\Delta m} \) follows Gauss distribution due to its dimension error. So the equivalent value is

\[
M(\Delta E_{\Delta m}) = \frac{E_{\Delta w} + E_{\Delta u}}{2}.
\]

(4)
\[
\begin{align*}
J_{E3} &= \frac{E_{\text{max}} + E_{\text{min}}}{2 \cos \alpha}, \\
D(J_{E3}) &= \left( \frac{E_{\text{max}} - E_{\text{min}}}{6 \cos \alpha} \right)^2.
\end{align*}
\] (5)

### 3.1.2 Gear radial integrated error

During the gear manufacturing process, geometry eccentricity will appear due to the existence of radial circle run-out. The influence of its radial component is similar to the one of the central distance deviation and obeys Gauss distribution. The equivalent value \(J_{E2}\) and variance \(D(J_{E2})\) of gear backlash caused by radial circle run-out of gear ring are

\[
\begin{align*}
J_{E2} &= 0, \\
D(J_{E2}) &= \left( \frac{\Delta F \tan \alpha}{3} \right)^2.
\end{align*}
\] (6)

### 3.1.3 Offset error

In Fig. 5, \(O_1\) and \(O_2\) are theoretical center points while \(O'_1\) is the actual gear center when offset error exists. \(P\) is the theoretical mesh point while \(P'\) is the actual mesh point. \(\Delta A\) is the value of offset error.

Assuming the normal offset as the basic line, the deviation \(\Delta f_a\) is symmetric with respect to the basic. While \(\Delta f_a\) is a size error, so it follows Gauss distribution too. The equivalent value \(J_{E3}\) and variance \(D(J_{E3})\) of gear backlash caused by radial circle run-out of gear ring are

\[
\begin{align*}
J_{E3} &= 0, \\
D(J_{E3}) &= \left( \frac{\Delta f_a \tan \alpha}{3} \right)^2.
\end{align*}
\] (7)

### 3.1.4 Integrated backlash for the first stage

The circumferential backlash calculated above is translational backlash. However, what we need is the angular backlash which is relative to the radius of reference circle. Therefore, each of the backlash should be converted to the output shaft and then added up. The equivalent value \(\Delta \varphi_{ab}\) and variance \(T \Delta \varphi_{ab}\) of return difference of the first stage are shown as

\[
\begin{align*}
\Delta \varphi_{ab} &= \frac{180 \times 60}{i_{\text{eff}}^2 \pi r_a} \sum_{i=1}^{J_{E1}} J_{E1}, \\
T \Delta \varphi_{ab} &= \frac{180 \times 60 T(j_E)}{i_{\text{eff}}^2 \pi r_a},
\end{align*}
\] (8)

where \(i_{\text{eff}}^2\) is the transmission ratio of whole system, and \(T(j_E)\) is the backlash tolerance of the first stage which can be represented by

\[
T(j_E) = \sqrt{\sum_{i=1}^{3} D(J_{Ei})}.
\] (9)

![Fig. 5 Offset error where \(O_1\) and \(O_2\) are theoretical center points while \(O'_1\) is the actual gear center when offset error exists, \(P\) is the theoretical mesh point while \(P'\) is the actual mesh point, and \(\Delta A\) is the value of offset error.]

Therefore, the return difference of output shaft caused by the first stage is

\[
\Delta \varphi_{ab} = \Delta \varphi_{ab} \pm \frac{T \Delta \varphi_{ab}}{2}.
\] (10)
3.2 Backlash analysis for the second stage

3.2.1 Inherent errors

Inherent errors of gear include the reduction of tooth thickness and gear eccentricity [16]. Reduction of tooth thickness is often represented by radial feeding error \( \Delta E_r \) of gear cutting and tooth thickness allowance \( s_E^\Delta \). Geometry eccentricity is often represented by radial circle run-out \( r_F^\Delta \) of gear ring gear or radial integrated error \( i_F^\Delta \) and motion eccentric is denoted by average deviation \( W_{ME}^\Delta \) of common normal line.

1) Tooth thickness allowance \( s_E^\Delta \)

Assuming the normal tooth thickness as a basic line, the upper deviation of tooth thickness is \( s_s^E \) and lower deviation is \( s_i^E \), and the tolerance is \( s_E^\Delta \) (Fig. 6). Both \( s_s^E \) and \( s_i^E \) should be minus due to gear backlash. \( s_E^\Delta \) is a type of size error while submits to Gauss distribution.

![Fig. 6 Tooth thickness allowance](image)

The equivalent value \( 1E_J \) and variance \( 1E_D^j \) of circumferential backlash caused by deviation of tooth thickness are

\[
\begin{align*}
    j_{E1} &= \frac{E_s^\Delta + E_i^\Delta}{2}, \\
    D(j_{E1}) &= \left(\frac{E_s^\Delta - E_i^\Delta}{6}\right)^2. 
\end{align*}
\]  

(11)

2) Average deviation of common normal line

The equivalent value \( j_E2 \) and variance \( D(j_{E2}) \) of circumferential backlash caused by average deviation of common normal line are

\[
\begin{align*}
    j_{E2} &= \frac{E_{w_m} + E_{w_m1}}{2\cos\alpha}, \\
    D(j_{E2}) &= \left(\frac{E_{w_m} - E_{w_m1}}{6\cos\alpha}\right)^2. 
\end{align*}
\]  

(12)

3) Radial circle run-out \( F_r^\Delta \) of gear ring

The equivalent value \( j_{E4} \) and variance \( D(j_{E4}) \) of circumferential backlash caused by radial circle run-out \( F_r^\Delta \) of gear ring are represented by

\[
\begin{align*}
    j_{E4} &= 0, \\
    D(j_{E4}) &= \left(\frac{\Delta F_r^\Delta K_n\tan\alpha}{3}\right)^2. 
\end{align*}
\]  

(13)

3.2.2 Assembly error

Offset error and pitch error of holes of box are included. However, only offset error is considered in this part because it has taken the pitch error of holes of box into account.

Offset error is the difference of practical center distance and normal center distance, which not only considers the pitch error of holes of box, but also takes the errors of each component that make up the gear pair into account.

The equivalent value \( j_{E5} \) and variance \( D(j_{E5}) \) of circumferential backlash caused by offset error are

\[
\begin{align*}
    j_{E5} &= 0, \\
    D(j_{E5}) &= \left(\frac{\Delta F_r^\Delta K_n\tan\alpha}{3}\right)^2. 
\end{align*}
\]  

(14)

3.2.3 Integrated backlash of the second stage

Based on the mathematical model of backlash, the equivalent value \( \Delta \phi_{i1} \) and variance \( T\Delta \phi_{i2} \) of return difference of the second stage can be obtained by

\[
\begin{align*}
    \Delta \phi_{i2} &= \frac{180 \times 60}{i_{i2}^2 \pi r_i} \sum_{i=1}^{k} j_{E2}, \\
    T\Delta \phi_{i2} &= \frac{180 \times 60T(j_E)}{i_{i2}^2 \pi r_i}. 
\end{align*}
\]  

(15)
where $i_{m2}^2$ is the transmission ratio of the second stage, and $T(j_e)$ is the backlash tolerance of the second stage which can be represented by

$$T(j_e) = \sqrt{\sum_{i=1}^{n} D(j_{m2})}. \tag{16}$$

Therefore, the return difference of output shaft caused by the second stage is

$$\Delta \phi_{l2} = \Delta \phi_{l2} \pm \frac{T \Delta \phi_{l2}}{2}. \tag{17}$$

### 3.3 Backlash analysis of bearing

#### 3.3.1 Radial clearance

If the maximum value of radial clearance $A_{max}$ and the minimum value $A_{min}$ submit to Gauss distribution, the equivalent value and variance can be obtained by

$$\begin{align*}
  j_{E1} &= \frac{A_{max} + A_{min}}{2}, \\
  D(j_{E1}) &= \left(\frac{A_{max} - A_{min}}{6}\right)^2.
\end{align*} \tag{18}$$

#### 3.3.2 Axial parallelism error of bearing

If the components of axial parallelism error of bearing with respect to the axis of plant gear in $x$ and $y$ are $\Delta f_x$ and $\Delta f_y$ respectively and obey Gauss distribution, then the equivalent value and its variance can be represented by

$$\begin{align*}
  j_{E2x} &= -\frac{1}{2} \times \frac{B}{b} \times f_x \tan \alpha, \\
  D(j_{E2x}) &= \left(\frac{1}{6} \times \frac{B}{b} \times f_x \tan \alpha\right)^2, \\
  j_{E2y} &= -\frac{1}{2} \times \frac{B}{b} \times f_y \tan \alpha, \\
  D(j_{E2y}) &= \left(\frac{1}{6} \times \frac{B}{b} \times f_y \tan \alpha\right)^2,
\end{align*} \tag{19}$$

and

$$\begin{align*}
  j_{E3x} &= -\frac{1}{2} \times \frac{B}{b} \times f_x \tan \alpha, \\
  D(j_{E3x}) &= \left(\frac{1}{6} \times \frac{B}{b} \times f_x \tan \alpha\right)^2,
\end{align*} \tag{20}$$

where $B$ is the length of bearing hole in planet gear in terms of mm, and $b$ is the width of gear in terms of mm.

#### 3.3.3 Backlash caused by bearing

Based on the mathematical models, the equivalent value $\Delta \phi_v$ and variance $T \Delta \phi_v$ of return difference of bearing can be represented by

$$\begin{align*}
  \Delta \phi_v &= \frac{180 \times 60 \sum_{i=1}^{n} j_{E2}}{\pi a_v}, \\
  T \Delta \phi_v &= \frac{180 \times 60 T(j_e)}{\pi a_v}, \tag{21}
\end{align*}$$

where $T(j_e)$ is the backlash tolerance of bearing and it can be shown as

$$T(j_e) = \sqrt{\sum_{i=1}^{n} D(j_{m2})}. \tag{22}$$

Therefore, the return difference of output shaft caused by bearing is

$$\Delta \phi_v = \Delta \phi_v \pm \frac{T \Delta \phi_v}{2}. \tag{23}$$

### 4 Computational results

The integrated backlash of the system can be obtained based on the above mathematical models. Errors affected the backlash of each stage are listed in Tables 2-4.

#### Table 2  Errors that affect backlash of the first stage

<table>
<thead>
<tr>
<th>Grade of accuracy</th>
<th>$\Delta E_{m1} / \mu m$</th>
<th>$\Delta \phi_{m1} / \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive gear</td>
<td>7-7-7HK</td>
<td>114 149 16.5 36</td>
</tr>
<tr>
<td>Driven gear</td>
<td>7-7-7HK</td>
<td>114 149 16.5 36</td>
</tr>
</tbody>
</table>

#### Table 3  Errors that affect backlash of the second stage

<table>
<thead>
<tr>
<th>Grade of accuracy</th>
<th>$\Delta E_{m2} / \mu m$</th>
<th>$\Delta E_{m2} / \mu m$</th>
<th>$\Delta \phi_{m2} / \mu m$</th>
<th>$\Delta F_{a} / \mu m$</th>
<th>$\Delta F_{a} / \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>External gear ring</td>
<td>7-7-7HK</td>
<td>112 168 128 192</td>
<td>7 36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal gear ring</td>
<td>7-7-7HK</td>
<td>112 168 128 192</td>
<td>7 36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4 Errors that affect backlash of bearing

<table>
<thead>
<tr>
<th></th>
<th>$\Delta A_{\text{min}}$/(\mu)m</th>
<th>$\Delta A_{\text{max}}$/(\mu)m</th>
<th>$f_x$/(\mu)m</th>
<th>$f_y$/(\mu)m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>2</td>
<td>13</td>
<td>5.5</td>
<td>11</td>
</tr>
</tbody>
</table>

According to the parameters above, the backlash of each stage and the whole system can be obtained. The results are shown in Table 5.

Table 5 Computational results

<table>
<thead>
<tr>
<th>Stage</th>
<th>Expression symbol</th>
<th>Computational result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st stage</td>
<td>$\Delta \theta_{\text{ab}}$</td>
<td>0.335 ± 0.103</td>
</tr>
<tr>
<td>2nd stage</td>
<td>$\Delta \theta_{12}$</td>
<td>0.419 ± 0.164</td>
</tr>
<tr>
<td>Bearing roller</td>
<td>$\Delta \theta_b$</td>
<td>0.6146 ± 0.335</td>
</tr>
<tr>
<td>The whole system</td>
<td>$\Delta \phi_b$</td>
<td>1.7557 – 0.9815</td>
</tr>
</tbody>
</table>

Table 5 shows that the backlash of the new reducer is a little large, of which the backlash of bearing takes a greater part and the backlash caused by the 2nd stage is smaller. Meanwhile, the backlash caused by the 1st stage is the smallest.

We presented some practical methods to reduce the backlash according to the results.

1) The crank bearing with small clearance should be chosen to improve the accuracy and reduce backlash;
2) Increase the dimensional size of external gear ring when the requirements for design is satisfied;
3) Increase the diameter of ring gear of the 1st stage without affecting the radial dimension of the reducer;
4) Improve the machine accuracy and assembly accuracy of components of the whole transmission system;
5) The transmission ratio should be as large as possible and be allocated reasonably.

5 Conclusions

We proposed a new planetary gearing with internal gear ring, which consists of two stages. The formulas of backlash of this kind of planetary drive are derived using probabilistic method. The prototype is manufactured based on the analysis of construction and working principle. The results show that

1) The most serious factor that affects the systematic backlash is the inherent error of gear, which can not be inevitable. However, improving the precision of machine tools and assembly accuracy can decrease the inherent error effectively;
2) The influences of each stage on backlash increase from the initial stage to the final one. That is, the backlash of the second stage has the greatest influence on the whole system, while the backlash caused by the initial stage has a smaller influence on the system. Therefore, we should try our best to control the backlash caused by the second stage;
3) Transmission ratio has great influence on the backlash of transmission chains, so the number of transmission stages should be reduced. The transmission ratio should be as large as possible and be allocated reasonably;
4) Backlash of gear train has a large relationship with ring gear. Therefore, the diameter of ring gear should be as large as possible if the requirements for design are satisfied.

References


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