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封闭截面夹芯杆的耦合变形和层间应力*

Coupling Deformation and Stresses Between Layers in Sandwich Bar with Closed Section

张培源　　　　巫祖烈
Zhang Peiyuan　　Wu Zulie
(重庆大学工程力学系,重庆,630044)

摘　要　用封闭截面薄壁杆理论建立的夹芯杆耦合变形的组合模型得到了芯层应力和层间结合应力的精确化分析方法,并给出了箱形截面杆的分析结果。

关键词　夹芯杆;封闭截面;耦合变形;层间应力
中国图书资料分类法分类号　TB124;TB125

ABSTRACT　According to the theory of thin-walled bar with closed section, a combined model is established of coupling deformation, and an accurate method of analysing stresses between layers is suggested. With the model, the analyses results were offered to sandwich bar with box section.

KEYWORDS　sandwich bar; closed section; coupling deformation; stresses between layers

0　引　　言

封闭截面夹芯杆由外表层柱壳、内表层柱壳和夹芯组成,见图1. 两表层由高模量与高强度材料制成,主要承受面内应力。夹芯主要承受剪切变形并阻止表层剥离。这类夹芯杆是构成复合材料梁的主要形式,已经得到广泛应用。但是,有关的分析方法[1~4]很少涉及耦合变形,很难涉及芯层应力和层间结合应力。芯层应力和层间应力正是控制多类失效形式的主要

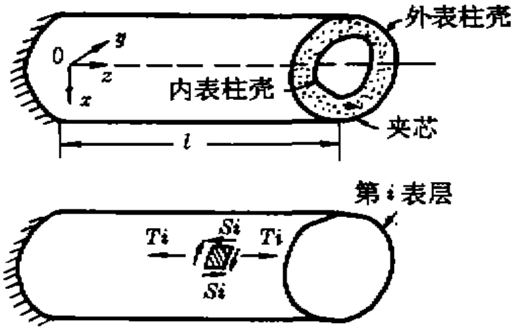

图 1　夹芯杆

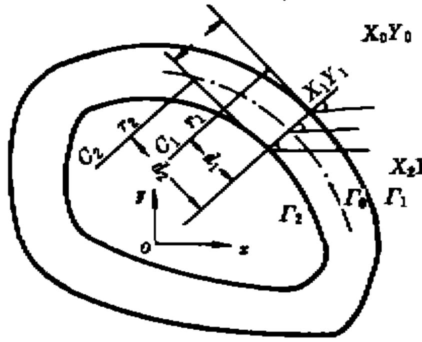

图 2　截面

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因素。本文用封闭截面单层薄壁杆理论建立夹芯杆耦合变形的组合模型，得到芯层和层间应力的精确化分析方法，给出了箱形截面杆的分析结果。

1 组合模型

讨论截面如图 2 所示的等截面直杆，截面几何参数和物理性质均与坐标 Z 无关。两表层厚度远小于夹芯厚度、表层扭转变形的刚周边假设和弯曲变形的平截面假设有效，因而表层位移分布为[1,4]：

$$u_i = d_i\theta_i + \xi\cos\alpha + \eta\sin\alpha,\ v_i = r_i\theta_i - \xi\sin\alpha + \eta\cos\alpha,\ w_i = \Omega_i\varphi_i\dot{\theta} + \zeta - \dot{\xi}x_i - \dot{\eta}y_i \qquad (1)$$

式中 i 取 1 和 2，分别表示外表层和内表层，

$$\varphi_i = \frac{s_i}{l_i} - \int_{(\Gamma_i)}^{s_i} r_i \mathrm{d}s_i / \Omega_i, \quad \Omega_i = \oint_{\Gamma_i} r_i \mathrm{d}s_i \qquad (2)$$

s_i 和 l_i 分别为表层截面中线 Γ_i 的弧长坐标和周长；θ_i 为表层截面扭转角；ξ、η、ζ 分别为杆截面的 x 向、y 向和轴向平动位移。这里假设这三个平动位移分量对内外表层取相等的值。对于每一表层，忽略周向正应力和周向正应变，根据式(1)，可得表层的纵向正应变和纵向一周向剪应变分别为

$$e_i = \dot{\zeta} - \ddot{\xi}x_i - \ddot{\eta}y_i + \Omega_i\varphi_i\ddot{\theta}, \quad \gamma_i = \Omega_i\dot{\theta}_i / l_i \qquad (3)$$

相应的内力用柱壳的本构方程表示为(图 1)

$$T_i = B_{11}^{(i)}e_i + B_{12}^{(i)}\gamma_i, \qquad S_i = B_{21}^{(i)}e_i + B_{22}^{(i)}\gamma_i \qquad (4)$$

式中 $B_{ij}^{(i)}$ 是第 i 表层柱壳的弹性系数，它们只是弧坐标 s_i 的函数，且 $B_{ij}^{(i)} = B_{ji}^{(i)}$. 式(1) 和(3) 中，上标“·”表示对 z 的导数。

对于芯层，忽略周向正应力、纵向正应力和周向 — 纵向剪应力，把其余 3 个应力分量表示为

$$\sigma_n = B_n\bar{e}_n, \quad \tau_{ns} = B_{ns}\bar{\gamma}_{ns}, \qquad \tau_{nz} = B_{nz}\bar{\gamma}_{nz} \qquad (5)$$

式中 B_n、B_{ns} 和 B_{nz} 是芯层材料的等效弹性系数，$\bar{e}_n$、$\bar{\gamma}_{ns}$ 和 $\bar{\gamma}_{nz}$ 为芯层厚度上的平均应变. 这样一来，将芯层的应力 σ_n、τ_{ns} 和 τ_{nz} 简化为厚度上均匀分布。为了将 $\bar{e}_n$、$\bar{\gamma}_{ns}$ 和 $\bar{\gamma}_{nz}$ 通过表层位移用 $\theta_1\theta_2\zeta\xi\eta$ 表示，我们列出用曲线坐标系 nsz 描写的应变 — 位移方程

$$e_n = \frac{\partial u}{\partial n}, \quad \gamma_{ns} = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial n} - \frac{u}{\rho}, \quad \gamma_{nz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial n}$$

式中 ρ 为坐标线 s 的曲率半径。计算其在芯层厚度上的均值，且分别取为

$$\bar{e}_n = \frac{1}{h_0}\int_{-h_0/2}^{h_0/2} e_n \mathrm{d}n, \quad \bar{\gamma}_{ns} = \frac{1}{h_0}\int_{-h_0/2}^{h_0/2} \gamma_{ns}\mathrm{d}n, \qquad \bar{\gamma}_{nz} = \frac{1}{h_0}\int_{-h_0/2}^{h_0/2} \gamma_{nz}\mathrm{d}n$$

计算中近拟地取

$$\frac{1}{h_0}\int_{-h_0/2}^{h_0/2} \frac{\partial u}{\partial s}\mathrm{d}n = \frac{1}{2}\left(\frac{\partial u_1}{\partial s_1} + \frac{\partial u_2}{\partial s_2}\right), \qquad \frac{1}{h_0}\int_{-h_0/2}^{h_0/2} \frac{v}{\rho}\mathrm{d}n = \frac{1}{2}\left(\frac{v_1}{\rho_1} + \frac{v_2}{\rho_2}\right),$$

$$\frac{1}{h_0}\int_{-h_0/2}^{h_0/2} \frac{\partial u}{\partial z}\mathrm{d}n = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_2}{\partial z}\right) = \frac{1}{2}(\dot{u}_1 + \dot{u}_2)$$

更假设芯层厚度 h_0 分段均匀，因而(图 2)

$$x_1 - x_2 \doteq h_0\cos\alpha, \quad y_1 - y_2 \doteq h_0\sin\alpha, \quad h \doteq h_0$$

于是得到

$$\bar{\varepsilon}_z = f_{z1}\theta_1 + f_{z2}\theta_2, \quad \bar{\gamma}_{sz} = f_{s1}\theta_1 + f_{s2}\theta_2, \quad \bar{\gamma}_{nz} = f_{n1}\dot{\theta}_1 + f_{n2}\dot{\theta}_2 \tag{6}$$

式中

$$\begin{aligned} &f_{z1} = d_1/h_0, \quad f_{z2} = -d_2/h_0, \\ &f_{s1} = \frac{1}{2}\frac{\mathrm{d}d_1}{\mathrm{d}s_1} + \frac{r_1}{h_0} - \frac{r_1}{2\rho_1}, \quad f_{s2} = \frac{1}{2}\frac{\mathrm{d}d_2}{\mathrm{d}s_2} - \frac{r_2}{h_0} - \frac{r_2}{2\rho_2}, \\ &f_{n1} = \frac{d_1}{2} + \frac{\Omega_1\varphi_1}{h_0}, \quad f_{n2} = \frac{d_2}{2} - \frac{\Omega_2\varphi_2}{h_0} \end{aligned} \tag{7}$$

式中 ρ_i 为表层截面中线 Γ_i 的曲率半径。

如果用 Γ_0 表示芯层截面中线，s_0 表示相应的曲线弧坐标，那末单位长度夹芯杆对应的应变能为

$$U = \sum_{i=1}^{2}\frac{1}{2}\oint_{\Gamma_i}(T_i e_i + s_i r_i)\mathrm{d}s_i + \frac{1}{2}\oint_{\Gamma_0}(\sigma_z\bar{\varepsilon}_z + \tau_{sz}\bar{\gamma}_{sz} + \tau_{nz}\bar{\gamma}_{nz})h_0\mathrm{d}s_0$$

把式(3)和(6)代入，总可以把 U 改写成

$$U = \frac{1}{2}(N\dot{\zeta} + M_y\ddot{\xi} - M_x\ddot{\eta} + G^{(1)}\ddot{\theta}_1 + G^{(2)}\ddot{\theta}_2 + H^{(1)}\dot{\theta}_1 + H^{(2)}\dot{\theta}_2 + M_z^{(1)}\theta_+ M^{(2)}Z\theta_2) \tag{8}$$

式中

$$\begin{aligned} &N = \sum_{i=1}^{2}\oint_{\Gamma_i}T_i\mathrm{d}s_i, \quad M_y = -\sum_{i=1}^{2}\oint_{\Gamma_i}T_i x_i\mathrm{d}s_i, \quad M_x = \sum_{i=1}^{2}\oint_{\Gamma_i}T_i y_i\mathrm{d}s_i, \\ &G^{(i)} = \Omega_i\oint_{\Gamma_i}T_i\varphi_i\mathrm{d}s_i, \quad H^{(i)} = \frac{\Omega_i}{l_i}\oint_{\Gamma_i}S_i\mathrm{d}s_i + \oint_{\Gamma_0}\tau_{nz}f_{ni}h_0\mathrm{d}s_0 \\ &M_z^{(i)} = \oint_{\Gamma_0}(\sigma_z f_{zi} + \tau_{sz}f_{si})h_0\mathrm{d}s_0 \end{aligned} \tag{9}$$

分别为与 ζ、ξ、η、$\bar{\theta}_i$、$\dot{\theta}_i$ 和 θ_i 相应的广义内力。

3　杆的等效本构方程

把 U 作为 $\ddot{\xi}$、$\ddot{\eta}$、$\ddot{\theta}_1$、$\ddot{\theta}_2$、$\dot{\zeta}$、$\dot{\theta}_1$、$\dot{\theta}_2$、θ_1 和 θ_2 共 9 个广义应变的 2 次齐次函数，按 Euler 定理，可以得到广义应变与它们对应的广义应力间的线性关系，即等效本构方程。一般而言，这 9 个方程可以得到简化。

特殊地选取坐标系 $oxyz$，使 ox 和 oy 两轴是第 i 表层的物理形心主轴，即对第 i 表层满足条件：

$$\oint_{\Gamma_i}B_{11}^{(i)}x_i\mathrm{d}s_i = \oint_{\Gamma_i}B_{11}^{(i)}y_i\mathrm{d}s_i = 0, \oint_{\Gamma_i}B_{11}^{(i)}x_i y_i\mathrm{d}s_i = 0 \tag{10}$$

其后的讨论中，更设两表层有相同的物理形心主轴。

总可以特殊地选择点 C_i 和计算弧点长坐标 s_i 的起点(图 2)，使

$$\oint_{\Gamma_i}B_{11}^{(i)}\varphi_i\mathrm{d}s_i = 0, \quad \oint_{\Gamma_i}B_{11}^{(i)}x_i\varphi_i\mathrm{d}s_i = \oint_{\Gamma_i}B_{11}^{(i)}y_i\varphi_i\mathrm{d}s_i = 0 \tag{11}$$

这样的点 C_i 就是第 i 表层的弯心。一般而言，两表层的弯心可以不得合。

对于厚度沿周向变化的夹芯杆，其弹性系数总满足条件

$$\beta_i = B_{12}^{(i)} / B_{11}^{(i)}, \qquad \mu_i = B_{22}^{(i)} / B_{11}^{(i)}, \qquad \beta_i、\mu_i = \text{const} \tag{12}$$

在上述条件下，得到的 9 个等效本构方程可以简化为彼此独立的如下 3 组

$$-M_y = A_{11}\ddot{\xi}, \quad M_x = A_{22}\ddot{\eta}; \quad G^{(1)} = A_{33}\ddot{\theta}_1, \qquad G^{(2)} = A_{44}\ddot{\theta}_2 \tag{13}$$

$$N = D_{11}\dot{\zeta} + D_{12}\dot{\theta}_1 + D_{13}\dot{\theta}_2, \quad H^{(1)} = D_{21}\dot{\zeta} + D_{22}\dot{\theta}_1 + D_{23}\dot{\theta}_2, \quad H^{(2)} = D_{21}\dot{\zeta} + D_{22}\dot{\theta}_1 + D_{23}\dot{\theta}_2 \tag{14}$$

$$M_z^{(1)} = C_{11}\theta_1 + C_{12}\theta_2, \qquad M_z^{(2)} = C_{21}\theta_1 + C_{22}\theta_2 \tag{15}$$

式中

$$\begin{aligned} A_{11} &= \sum_{i=1}^{2} \oint_{\Gamma_i} B_{11}^{(i)} x_i^2 \mathrm{d}s_i, \qquad A_{22} = \sum_{i=1}^{2} \oint B_{11}^{(i)} y_i^2 \mathrm{d}s_i, \\ A_{33} &= \Omega_1^2 \oint_{\Gamma_1} B_{11}^{(1)} \varphi_1^2 \mathrm{d}s_1, \qquad A_{44} = \Omega_2^2 \oint_{\Gamma_2} B_{11}^{(2)} \varphi_2^2 \mathrm{d}s_2 \end{aligned} \tag{16}$$

$$\begin{aligned} D_{11} &= \sum_{i=1}^{2} \oint B_{11}^{(i)} \mathrm{d}s_i, \qquad D_{12} = \frac{\Omega_1}{l_1} \oint_{\Gamma_1} B_{12}^{(1)} \mathrm{d}s_1, \qquad D_{13} = \frac{\Omega_2}{l_2} \oint_{\Gamma_2} B_{12}^{(2)} \mathrm{d}s_2 \\ D_{21} &= D_{12}, \quad D_{22} = \Omega_1^2 \frac{1}{l_1^2} \oint_{\Gamma_1} B_{22}^{(1)} \mathrm{d}s_1 + \oint_{\Gamma_0} f_{x1}^2 B_{ss} h_0 \mathrm{d}s_0, \quad D_{23} = D_{32} \end{aligned} \tag{17}$$

$$D_{31} = D_{13}, \quad D_{32} = \oint_{\Gamma_0} f_{x1} f_{x2} B_{ss} h_0 \mathrm{d}s_0, \qquad D_{33} = \frac{\Omega_2^2}{l_2^2} \oint_{\Gamma_2} B_{22}^{(2)} \mathrm{d}s_2 + \oint_{\Gamma_0} f_{x2}^2 B_{ss} h_0 \mathrm{d}s_0,$$

$$C_{11} = \oint_{\Gamma_0} (f_{s1}^2 B_s + f_{x1}^2 B_{ss}) h_0 \mathrm{d}s_0, \qquad C_{12} = C_{21} = \oint_{\Gamma_0} (f_{s1} f_{s2} B_s + f_{x1} f_{x2} B_{ss}) h_0 \mathrm{d}s_0 \tag{18}$$

$$C_{22} = \oint_{\Gamma_0} (f_{s2}^2 B_{11} + f_{x2}^2 B_{ss}) h_0 \mathrm{d}s_0$$

4　控制方程和边界条件

如果表层柱壳上分布外力的 3 个分量分别为 $q_1^{(i)}$、$q_2^{(i)}$ 和 $q_3^{(i)}$，端面 $z = 0$、L 上表层柱壳内力的边值为 $\check{T}$ 和 $\check{S}_i$，夹芯不受外力。那末，单位长度杆对应的外力势为

$$F = \sum_{i=1}^{2} \oint_{\Gamma_i} (q_1^{(i)} u_i + q_2^{(i)} v_i + q_3^{(i)} w_i) \mathrm{d}s_i + \sum_{i=1}^{2} \oint_{\Gamma_i} \{[\check{S}_i v_i] + [\check{T}_i w_i]\} \mathrm{d}s_i$$

式中约定

$$[\check{S}_i v_i] = (\check{S}_i v_i)_{z=L} - (\check{S}_i v_i)_{z=0}, \qquad [\check{T}_i w_i] = (\check{T}_i w_i)_{z=L} - (\check{T}_i w_i)_{z=0} \tag{19}$$

把式(1) 代入，分部积分，得到

$$\begin{aligned} F = {} & P_z\zeta + P_x\xi + P_y\eta + m^{(1)}\theta_1 + m^{(2)}\theta_2 + [\check{N}\zeta] + [\check{M}_y\xi'] - \check{M}_x\eta'] + \\ & [\check{Q}_x\xi] + [\check{Q}_y\eta] + [\check{G}^{(1)}\theta_1] + [\check{G}^{(2)}\theta_2] + [\check{H}^{(1)}\theta_1] + [\check{H}^{(2)}\theta_2] \end{aligned} \tag{20}$$

式中

$$\left.\begin{aligned} P_z &= \sum_{i=1}^{2} \oint_{\Gamma_i} q_3^{(i)} \mathrm{d}s_i, \qquad P_x = \sum_{i=1}^{2} \oint_{\Gamma_i} (q_1^{(i)} \cos\alpha - q_2^{(i)} \sin\alpha + x_i q_3^{(i)\prime}) \mathrm{d}s_i \\ P_y &= \sum_{i=1}^{2} \oint_{\Gamma_i} (q_1^{(i)} \sin\alpha + q_2^{(i)} \cos\alpha + y_i q_3^{(i)\prime}) \mathrm{d}s_i \qquad m^{(1)} = \oint_{\Gamma_1} (q_1^{(1)} d_1 + q_2^{(1)} r_1 - \Omega_1 \varphi_1 q_3^{(1)\prime}) \mathrm{d}s_1 \\ m^{(2)} &= \oint_{\Gamma_2} (q_1^{(2)} d_1 - q_2^{(2)} r_2 - \Omega_2 \varphi_2 q_3^{(2)}) \mathrm{d}s_2 \end{aligned}\right\}$$

$$\left.\begin{aligned}
&\check N = \sum_{i=1}^{2}\oint_{\Gamma_i}\check T_i \mathrm{d}s_i, \quad \check M_y = -\sum_{i=1}^{2}\oint_{\Gamma_i}\check T_i x_i \mathrm{d}s_i, \quad \check M_x = \sum_{i=1}^{2}\oint_{\Gamma_i}\check T_i y_i \mathrm{d}s_i \\
&\check Q_x = -\sum_{i=1}^{2}\oint_{\Gamma_i}(\check S_i \sin\alpha + \dot q_3^{(i)} x_i)\mathrm{d}s_i, \quad \check Q_y = \sum_{i=1}^{2}\oint_{\Gamma_i}(\check S_i \sin\alpha - \dot q_3^{(i)} y_i)\mathrm{d}s_i \\
&\check G^{(i)} = \Omega_i \oint_{\Gamma_i}(\check T_i \varphi_i \mathrm{d}s_i, \quad \check H^{(i)} = \Omega_i \oint_{\Gamma_i}\check S_i r_i \mathrm{d}s_i + \dot G^{(i)}
\end{aligned}\right\} \tag{21}$$

这里 $\dot G^{(i)} = (G^{(i)})_{z=0,L}$.

按最小势能原理

$$\delta\int_0^L (U - F)\mathrm{d}z = 0 \tag{22}$$

得到它的 Euler 方程和边值条件：

$$-\ddot M_y + P_x = 0, \quad [(M_y - \check M_y)\delta\dot\xi] = 0, \quad [(\dot M_y + \check Q_x)\delta\xi] = 0 \tag{23}$$

$$\ddot M_x + P_y = 0, \quad [(M_x - \check M_x)\delta\dot\eta] = 0, \quad [(\dot M_x - \check Q_y)\delta\eta] = 0 \tag{24}$$

$$\dot N + P_z = 0, \quad [(N - \check N)\delta\zeta] = 0 \tag{25}$$

$$\begin{aligned}
&\ddot G^{(i)} - \dot H^{(i)} + M_z^{(i)} + m^{(i)} = 0, [(G^{(i)} - \check G^{(i)})\delta\dot\theta_i] = 0, \\
&[(H^{(i)} - \dot G^{(i)} - \check H^{(i)})\delta\theta_i] = 0
\end{aligned} \tag{26}$$

把式(13)(14)和(15)代入，便得到用 ζ、ξ、η、θ_1 和 θ_2 描写的控制方程和边界条件。这种用广义位移表示的边值问题可以分为 3 个彼此独立的问题。

zoz 面内的弯曲

$$A_{11}\ddddot\xi + P_x = 0, \quad [(A_{11}\ddot\xi + \check M_y)\delta\dot\xi] = 0, \quad [(-A_{11}\dddot\xi + \check Q_x)\delta\xi = 0 \tag{27}$$

yoz 面内的弯曲

$$A_{22}\ddddot\eta + P_y = 0, \quad [(A_{22}\ddot\eta - \check M_x)\delta\dot\eta] = 0, \quad [(-A_{22}\dddot\eta + \check Q_y)\delta\eta = 0 \tag{28}$$

内外表层扭转和轴向拉压耦合

$$\begin{aligned}
&D_{11}\ddot\zeta + D_{12}\dot\theta_1 + D_{13}\dot\theta_2 + P_z = 0 \\
&A_{33}\ddddot\theta - D_{22}\ddot\theta - D_{23}\ddot\theta_2 + C_{11}\theta_1 + C_{12}\theta_2 - D_{21}\dot\zeta + m^{(1)} = 0 \\
&A_{44}\ddddot\theta - D_{33}\ddot\theta - D_{32}\ddot\theta_2 + C_{21}\theta_1 + C_{22}\theta_2 - D_{31}\dot\zeta + m^{(2)} = 0
\end{aligned} \tag{29}$$

在特殊情况下，$B_{12}^{(i)} = 0, i = 1,2$，那末轴向变形和表层扭转变形不耦合，$D_{12} = D_{21} = D_{13} = D_{31} = 0$. 但两表层的扭转变形总耦合在两个常微分方程中。

5　箱形截面杆的弹性系数和层间应力

讨论图 3 所示箱形截面，内外表层和芯层都是匀质材料，截面几何图形有彼此正交的两对称轴一轴 ox 与轴 oy. 两表层的物理形心和弯心都与点 O 重合。在 $x = \pm a_1$，外表层厚度为常值 h_1，在 $y = \pm b_1$ 外表层厚度为常值 $\delta_1 h_1$；在 $x = \pm a_2$，内表层厚度为常值 h_2，在 $y = \pm b_2$，内表层度为常值 $\delta_2 h_2$；在 $x = \pm a_0$

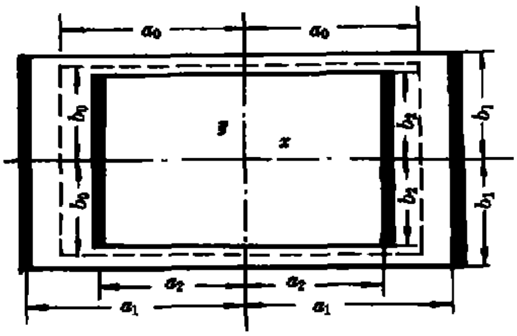

图 3　箱形截面

芯层厚度为 h_0，在 $y=\pm b_0$，芯层厚度为 $\delta_0 h_0$。用 $B^{(i)}$ 表示 $B_{11}^{(i)}$ 在 $x=\pm a_i$ 的取值，因此 $y=\pm b_i$ 处 $B^{(i)}=\delta_i B^{(i)}$. 芯层等效弹性常数 B, B_{44}, B_{55} 保持常值。

式(2)给出的 φ_i 为

$$\varphi_i=-\frac{a_i-b_i}{8b_i(a_i+b_i)}y,\qquad x=\pm a_i;\qquad \varphi=-\frac{a_i-b_i}{8a_i(a_i+b_i)}x,\qquad y=\pm b_i$$

式(7)给出的 f_{ri}、f_{si} 和 f_{xi}

$$f_{r1}=\begin{cases}-y/h_0 & x=\pm a_0\\ -x/h_0 & y=\pm b_1\end{cases}\qquad f_{r2}=\begin{cases}y/h_0 & x=\pm a_0\\ x/h_0 & y=\pm b_0\end{cases}$$

$$f_{s1}=\begin{cases}\dfrac{a_1}{h_0}-\dfrac{1}{2} & x=\pm a_0\\ \dfrac{b_1}{h_0}-\dfrac{1}{2} & y=\pm b_0\end{cases}\qquad f_{s2}=\begin{cases}-\dfrac{a_2}{h_0}-\dfrac{1}{2} & x=\pm a_0\\ -\dfrac{b_2}{h_0}-\dfrac{1}{2} & y=\pm b_0\end{cases}$$

$$f_{x1}=\begin{cases}-\left(\dfrac{a_1(a_1-b_1)}{h_0(a_1+b_1)}+\dfrac{1}{2}\right)y & x=\pm a_0\\ -\left(\dfrac{b_1(a_1-b_1)}{h_0(a_1+b_1)}-\dfrac{1}{2}\right)x & y=\pm b_0\end{cases}$$

$$f_{x2}=\begin{cases}\left(\dfrac{a_2(a_2-b_2)}{h_0(a_2+b_2)}-\dfrac{1}{2}\right)y & x=\pm a_0\\ \left(\dfrac{b_2(a_2-b_2)}{h_0(a_2+b_2)}-\dfrac{1}{2}\right)x & y=\pm b_0\end{cases}$$

式(16)、(17)和(18)给出

$$A_{11}=4\sum_{i=1}^{2}a_i^2\left(b_i+\frac{1}{3}a_i\delta_i\right)B^{(i)},\qquad A_{22}=4\sum_{i=1}^{2}b_i^2\left(a_i\delta_i+\frac{1}{3}b_i\right)B^{(i)}$$

$$A_{33}=\frac{4}{3}a_1^2b_1^2(a_1-b_1)\psi_1B^{(1)},\qquad A_{44}=\frac{4}{3}a_2^2b_2^2(a_2+b_2)\psi_2B^{(2)}$$

$$D_{11}=4\sum_{i=1}^{2}(a_i\delta_i+b_i)B^{(i)},\qquad D_{12}=D_{21}=\frac{8\beta_1a_1b_2}{a_1+b_1}(a_1\delta_1+b_1)B^{(1)}$$

$$D_{13}=D_{31}=\frac{8\beta_2a_2b_2}{a_2+b_2}(a_2\delta_2+b_2)B^{(2)}$$

$$D_{22}=\frac{8\gamma_1a_1^2b_1^2}{(a_1+b_1)^2}(a_1\delta_1+b_1)B^{(1)}+\frac{4}{3}B_{44}h_0\left[\left(\frac{a^1(a_1-b_1)}{h_0(a_1+b_1)}+\frac{1}{2}\right)^2b_0^3+\left(\frac{b_1(a_1-b_1)}{\delta_0h_0(a_1+b_1)}+\frac{1}{2}\right)^2a_0^3\right]$$

$$D_{33}=\frac{8\gamma_2a_2^2b_2^2}{(a_2+b_2)^2}(a_2\delta_2+b_2)B^{(2)}+\frac{4}{3}B_{44}h_0\left[\left(\frac{a^2(a_2-b_2)}{h_0(a_2+b_2)}-\frac{1}{2}\right)^2b_0^3+\left(\frac{b_2(a_2-b_2)}{\delta_0h_0(a_2+b_2)}-\frac{1}{2}\right)^2a_0^3\right\}$$

$$D_{32}=D_{23}=-\frac{4}{3}B_{44}h_0\left[\left(\frac{a_1(a_1-b_1)}{h_0(a_1+b_1)}+\frac{1}{2}\right)\left(\frac{a_2(a_2-b_2)}{h_0(a_2+b_2)}-\frac{1}{2}\right)b_0^3+\left(\frac{b_1(a_1-b_1)}{\delta_0h_0(a_1+b_1)}+\frac{1}{2}\right)\left(\frac{a^2(a_2-b_2)}{h_0(a_2+b_2)}-\frac{1}{2}\right)a_0^3\right]$$

$$C_{11}=\frac{4}{3}B_s\frac{1}{h}\left(b_0^3+\frac{a_0^3}{\delta_0}\right)+4B_sh_0\left[\left(\frac{a_1}{h_0}-\frac{1}{2}\right)^2b_0+\delta_0\left(\frac{b_1}{\delta_0h_0}-\frac{1}{2}\right)^2a_0\right]$$

$$C_{22}=\frac{4}{3}B_s\frac{1}{h}\left(b_0^3+\frac{a_0^3}{\delta_0}\right)+4B_sh_0\left[\left(\frac{a_1}{h_0}+\frac{1}{2}\right)^2b_0+\delta_0\left(\frac{b_1}{\delta_0h_0}+\frac{1}{2}\right)^2a_0\right]$$

$$C_{12}=C_{21}=-\frac{4}{3}B_s\frac{1}{h_0}\left(b_0^3+\frac{a_0^3}{\delta_0}\right)-$$

$$-4B_sh_0\left[\left(\frac{a_1}{h_0}-\frac{1}{2}\right)\left(\frac{a_2}{h_0}+\frac{1}{2}\right)b_0+\delta_0\left(\frac{b_1}{\delta_0h_0}-\frac{1}{2}\right)\left(\frac{b_1}{\delta_0h_0}+\frac{1}{2}\right)a_0\right]$$

式中

$$\psi_i=(8e_i-1)^2+8\left[\left(\frac{1}{4}-e_i\right)-1\right]^2\left(\frac{1}{4}-e_i\right)\delta_i,\quad e_i=b_i/[4(a_i+b_i)],i=1,2$$

按照式(5)和(6)，B_sf_{si} 和 $B_{si}f_{ni}$ 分别为扭角 θ_i 的单位值产生的芯层应力 σ_s 和 τ_{ns}，$B_{si}f_{ri}$ 为扭率 θ_i' 的单位值产生的芯层应力 τ_{nz}。图 4 表示了 f_{si}、f_{ni} 和 f_{ri} 在 Γ_0 上的分布规律。由于芯层应力取为沿厚度均匀分布，应力分量 σ、τ_{ns}、τ_{nz} 就是外表与芯层和内表与芯层间的层间应力。

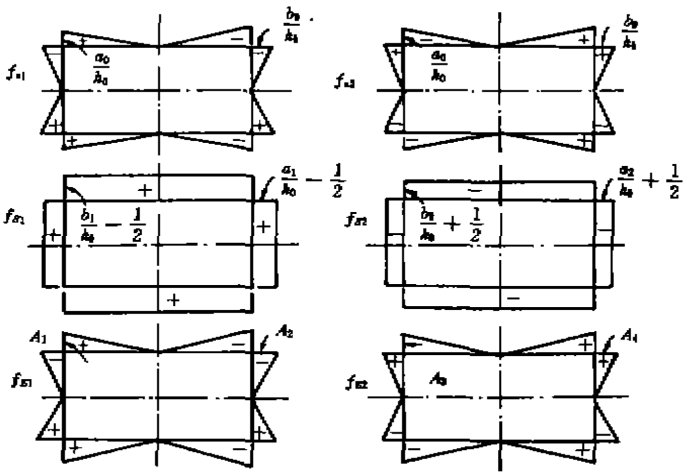

图 4　f_{si}、f_{ni}、f_{ri} 在 Γ_0 上的分布

$$A_1=\left(\frac{b_1(a_1-b_1)}{h_0(a_1+b_1)}+\frac{1}{2}\right)a_0\qquad A_3=\left(\frac{b_2(a_2-b_2)}{h_0(a_2+b_2)}-\frac{1}{2}\right)a_0$$

$$A_2=\left(\frac{a_1(a_1-b_1)}{h_0(a_1+b_1)}+\frac{1}{2}\right)b_0\qquad A_3=\left(\frac{a_2(a_2-b_2)}{h_0(a_2+b_2)}-\frac{1}{2}\right)b_0$$

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