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Applications of Nonlinear Schrödinger Equations in Optical Fiber Telecommunication Systems*

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ABSTRACT: The paper presents comprehensively some applications of the nonlinear Schrödinger (NLS) equations in the optical fiber telecommunication systems. The main contents are presented in the following six sections: 1) N-soliton interaction in optical fibers; 2) pulse propagation in the fiber; 3) the femtosecond soliton-soliton interaction in dispersion-shifted fiber; 4) strongly coupled nonlinear parametric solitary waves; 5) higher-order effects in bandwidth-limited soliton propagation in optical fibers; 6) suppression of noise accumulation in bandwidth-limited soliton transmission by means of linear loop mirrors.

KEYWORDS: telecommunication system; nonlinear Schrödinger equation; optical fiber; solitary wave; soliton

Under what conditions the dynamics of the subharmonic field are described by a nonlinear Schrödinger equation is presented. We also present new solitary-wave solutions to the traveling-wave parametric amplifier equations for arbitrary phase matching in which there are equal group velocities and no pump dispersion. In this strongly coupled case one condition for solitary-wave propagation is that the subharmonic field obeys a nonlinear Schrödinger equation while the pump field is proportional to the square of the subharmonic. The interesting feature of the new solutions is that they are valid even for exact phase matching, in which the effective nonlinearity is very large. These results show that the two coupled partial differential equations describing traveling-wave parametric amplifiers encompass a broader class of solitary-wave solutions than the nonlinear Schrödinger equation.

Solitons and fiber amplifiers will be essential components of future ultrahigh-bandwidth optical telecommunication systems. Subpicosecond (10^{-12} s)-to-femtosecond(10^{-15} s) soliton pulses are considered to be very attractive candidates as signal carriers for realizing an ultrahigh-speed optical telecommunication system at a few hundred gigabits(10^9 bits) per second. For femtosecond soliton telecommunication to be achieved, it is important to understand the characteristics of solitons when they are transmitted through an optical fiber. These include soliton-soliton interaction, the soliton self-frequency shift(SSFS), and the effect of polarization on the single-wavelength capacity of a soliton transmission system and has permitted 40 G bit/s doubling of the single-wavelength capacity of a soliton transmission system and has permitted 40 G bit/s transmission over 1000 km[1]. If polarization-multiplexed soliton pulses in the femtosecond regime are used, there is a possibility of further increasing the bit rate into the terabit(10^{12} bits)-per-second regime.

1 N-soliton interaction in Optical Fibers

L. Gagnon and N. Sitévenar[2] present an exact solution of the nonlinear Schrödinger equation that describes various interactions of N pulses of equal amplitudes.

Two neighboring temporal(or spatial) solitons in nonlinear optics are known to suffer mutual interaction that depends on their relative amplitude, phase, group velocity, and initial distance. Even in the more practical situation in which the amplitudes and group velocities are the same for both solitons, they experience interaction that tends to attract or

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repulse them, depending on their relative phase and initial distance. This phenomenon is an intrinsic property of the nonlinear Schrödinger (NLS) equation

$$iu_x + \alpha u_{tt} + u \mid u \mid^2 = 0 \quad (1)$$

which provides the well-known basic model for model for the self-modulation (or self-focusing) theory[3], where $u(t, x)$ is the amplitude of the electric field, x is the propagation coordinate, and t is the reduced time coordinate.

Numerical simulations of 2-and N-soliton interactions have trains[4,5]. Soliton interaction has also been studied analytically with the general framework of the inverse-scattering-transform (IST) method[6]. Various perturbation approaches have also been used, mainly in the regime of weakly overlapping pulses[7,8]. The main result of these analyses was that the interaction force between two solitons of equal amplitude decreases exponentially with the initial distance between them. The force is attractive when the two solitons are in phase and leads to a periodic collapse of the two solitons; the force is repulsive when the two initial solitons are out of phase so that the solitons eventually separate, even for a relatively small initial phase difference. Experimental observations have also confirmed these properties, at least within a range in which the self-stimulated Raman scattering does not destroy the periodic collapse[9].

L. Gagnon et al. present some exact solutions of the NLS equation that describes the interaction between N solitons of equal amplitude but with a particular chirp. Here the number N refers to the multiplicity of one unique pole.

The solutions are obtained by solving of the NLS equation using the so-called dressing method which transforms the problem into Zakharov-Shabat linear differential matrix system

$$F_t = \mathbf{U}F, \qquad F_x = \mathbf{V}F$$

where

$$\mathbf{U} = i\begin{bmatrix} -\lambda/2 & u^* \\ u & \lambda/2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} -i \mid u \mid^2 + i\lambda^2/2 & u_t^* - i\lambda u^* \\ -u_t - i\lambda u & i \mid u \mid^2 - i\lambda^2/2 \end{bmatrix}$$

and λ is a complex parameter.

2 Pulse Propagation in the Fiber

W. Forysiak et al.[10] studied average soliton propagation in periodically amplified soliton systems with stepwise varying fiber dispersion. Optimal dispersion profiles are obtained by minimization of a simple and intuitively appealing perturbation function. At a given data rate, n-fold profiling is shown to permit n-fold extension of amplifier spacing. Equivalently, at a given amplifier spacing, it permits $\sqrt{n}$-fold increase of data rate. They demonstrated the feasibility of 40-G bit/s operation over several hundred km, using 5-ps solitons and 50-km amplifier spans.

In periodically amplified soliton transmission systems, pertubations to the data stream occur because of the addition of spontaneous emission noise at the amplifiers[11] and because of a discontinuous balance between dispersion and nonlinearity over each amplification period[12].

At higher data rates over short transmission distances, the effects of concatenated noise are less restrictive, and the average soliton condition becomes the dominant constraint on data capacity. Average soliton perturbations rapidly dominate high-data-rate system design.

Pulse propagation in the fiber is described by the normalized nonlinear Schrödinger equation[12].

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + \mid u \mid^2 u = -i\Gamma u \quad (3)$$

where u represents the electric-field amplitude and Γ represents the fiber loss. The amplifier power gains are $G = \exp(2\Gamma_{z_a})$, where z_a is the normalized amplifier spacing. After introduction of the transformation $u(z, x) = \alpha(z)A(z, t)$, Eq. (3) becomes

$$i\frac{\partial A}{\partial z} + \frac{1}{2}\frac{\partial^2 A}{\partial t^2} + \alpha^2(z) \mid A \mid^2 A = 0 \quad (4)$$

where $\alpha^2(z) = U(z) = A_0^2\exp(-2\Gamma z)$ and

$$A_0^2 = \frac{2\Gamma z_a}{1 - \exp(-2\Gamma z_a)} \quad (5)$$

for $\langle U(z)\rangle = 1$. Thus the exponential energy decay in the periodically nonlinear Schrödinger equation is equivalent to an exponential decay, $U(z)$, in the nonlinear coefficient of the lossless nonlinear Schrödinger equation. If the period of $U(z)$ is short compared with the soliton period, its average is a good approximation in Eq. (4), and may obtain the average soliton model[12,13], where A_0 is the $N=1$ soliton amplitude in Eq. (3)

3 the Femtosecond Soliton-soliton Interaction in Dispersion-shifted Fiber

T. Sugawa et al.[14] had investigated the polarization dependence of femtosecond soliton-soliton interaction in a dispersion-shifted fiber. To analyze the femtosecond soliton-soliton interaction in a dispersion-shifted fiber, they simulated the coupled nonlinear Schrödinger equations, given by

$$-i\left(\frac{\partial u}{\partial q}+\delta\frac{\partial u}{\partial t}\right)=\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u+\frac{2}{3}|v|^2u+\frac{1}{3}v^2u^*\exp(-iR\delta q)+i\frac{\partial u}{\partial q}\Big|_{Raman} \quad (6a)$$

$$-i\left(\frac{\partial v}{\partial q}-\delta\frac{\partial v}{\partial t}\right)=\frac{1}{2}\frac{\partial^2 v}{\partial t^2}+|v|^2v+\frac{2}{3}|u|^2v+\frac{1}{3}u^2v^*\exp(-iR\delta q)+i\frac{\partial v}{\partial q}\Big|_{Raman} \quad (6b)$$

where

$$i\frac{\partial u}{\partial q}\Big|_{Raman}=-\left(c_1u\frac{\partial|u|^2}{\partial t}+c_2u\frac{\partial|v|^2}{\partial t}+c_3v\frac{\partial uv^*}{\partial t}\right) \quad (7a)$$

$$i\frac{\partial v}{\partial q}\Big|_{Raman}=-\left(c_1v\frac{\partial|v|^2}{\partial t}+c_2v\frac{\partial|u|^2}{\partial t}+c_3u\frac{\partial u^*v}{\partial t}\right) \quad (7b)$$

u and v are orthogonally polarized solitons. R is the normalized wave vector, and δ is the walk-off between polarization modes. The fourth terms on the right-hand sides of eqs. (6) indicate a coherent interaction, which is included in the simulation. Since the birefringence of the fiber is weak, the phase-rotation term moves slowly compared with that of highly birefringent fiber; therefore $\exp(-iR\delta q)$ is not averaged out to zero. Thus the coherent term should be taken into account in their numerical analysis.

4 Strongly Coupled Nonlinear Parametric Solitary Waves

M. J. Werner et al.[15] derived conditions under which solitary simulton waves can propagation $X^{(2)}$ parametric waveguides. These conditions can occur both with and without the phase matching.

Introduce new solitary-wave solutions for equal group velocity and zero pump dispersion, in which the two fields have different temporal profiles. These new results extend the domain of the Schrödinger-like solutions, extended to hold only for large phase mismatching. The problem of finding solitary-wave solutions reduces directly to a nonlinear Schrödinger equation in φ. Suppose that Ψ is solitary, with $\Psi\sim\exp(i\Omega\xi)$; then $\partial\Psi/\partial\xi=i\Omega\Psi$, so that

$$\Psi=\varphi^2/2i(\beta-\Omega) \quad (8)$$

It follows immediately that φ must vary as $\exp(i\Omega\xi/2)$ and satisfies

$$\left[i\frac{\partial}{\partial\xi}-\frac{1}{2}\mathrm{sgn}(k''_2)\frac{\partial^2}{\partial\tau^2}-\frac{1}{2(\beta-\Omega)}|\varphi|^2\right]\varphi=0 \quad (9)$$

which is similar to the nonlinear Schrödinger equation obtained above. However, Eq. (9) doesn't imply that any N-soliton solution for φ is a coupled soliton of the full equations; only solutions of the form $\varphi\sim\exp(i\Omega\xi/2)$ are compatible with their assumptions. Since β can have either sign, bright solitary waves exist for both anomalous and normal dispersions. It is also for dark solitary solutions. Particular solitary-wave solutions for which $\Omega=-\gamma^2\,\mathrm{sgn}(k''_2)$ are

$$\varphi=[2\gamma^2\mathrm{sgn}(k''_1)(\beta-\Omega)]^{1/2}\mathrm{sech}(\gamma\tau)\cdot\exp\left[-i\frac{\gamma^2}{2}\mathrm{sgn}(k''_1)\xi\right] \quad (10a)$$

$$\Psi=-i\gamma^2\mathrm{sgn}(k''_1)\mathrm{sech}^2(\gamma\tau)\cdot\exp[-i\gamma^2\mathrm{sgn}(k''_1)\xi] \quad (10b)$$

which are bright solitary waves for $\mathrm{sgn}(k''_1)(\beta-\Omega)>0$, and

$$\varphi=[2\gamma^2\mathrm{sgn}(k''_1)(\beta-\Omega)]^{1/2}\tanh(\gamma\tau)\cdot\exp\left[i\frac{\gamma^2}{2}\mathrm{sgn}(k''_1)\xi\right] \quad (11a)$$

$$\Psi=i\gamma^2\mathrm{sgn}(k''_1)\tanh^2(\gamma\tau)\cdot\exp[i\gamma^2\mathrm{sgn}(k''_1)\xi] \quad (11b)$$

in this case of dark solitary waves for $\mathrm{sgn}(k''_1)(\beta-\Omega)<0$. Note that for the bright solitary waves, the energy scales with γ^2 for $|\beta|<\gamma^2$, which can be much smaller than in the phase-mismatched solution. In summary, the new solutions permit the possibility of the formation of solitary waves at a much lower intensity than in the case of large phase mismatching.

5 Higher-order Effects in Bandwidth-limited Soliton Propagation in Optical Fibers

A. B. Aceves et al.[16] examined the effects of higher-order linear and nonlinear terms in bandwidth-limited amplified soliton propagation using numerical studies and soliton perturbation theory. These effects are responsible for strong reduction of soliton-soliton interaction in such systems.

The insertion of bandpass filters after the lumped amplification was first proposed in long-distance transmission links to reduce noise-induced temporal jitter in soliton propagation[17]. It was then recognized that bandwidth-limited amplification (BLA) has other positive implications both in transmission links and in fiber lasers such as the reduction of soliton-soliton interaction[18,19] and of collision-induced temporal jitter in wavelength-division-multiplexing transmission[20].

The so-called complex Ginzburg-Landau equation, a generalization of the nonlinear Schrödinger equation with complex coefficients and including a linear amplification term, has been considered to describle pulse propagation fibers with BLS. As is well known, however, a correct model must take into account higher-order linear and nonlinear effects[21], particularly but not exclusively for pulses in the subpicosecond range, such as those now commonly generated in a fiber laser.

We focus our attention on the role played by nonlinear effects, i. e., nonlinear dispersion (NLD) and intrapulse Raman scattering (IRS). If BLA is absent , NLD, which is caused by the intensity dependence of the group velocity, is responsible for asymmetric spectral broadening and self-steepening of the trailing edge of the pulse[22,23]. IRS causes a continuous downshift of the carrier frequency of the pulse, known as self-frequency shift. This self-frequency shift also originates an acceleration that adds an extra velocity component to the one that is due to NLD. This effect can be suppressed by BLA since the filter introduces losses for the downshifted frequencies[24,25].

Recently extensive studies on the effects of third-order linear dispersion (TOD) in a BLA system were presented[26,27]. The authors analyzed the propagation of solitons in a BLA system with NLD, and IRS using numerical studies and soliton perturbation theory. A comparison with TOD-induced effects reveals that IRS is of the same order or even stronger in reducing soliton-soliton interaction.

Propagation of short light pulses in optical fibers in the presence of BLA can be decribed by a modified complex Ginzburg-Landau equation or, equivalently in the sense previously specified, by a perturbed nonlinear Schrodinger equation[21]:

$$q_z = \frac{i}{2} q_{tt} + i \quad q |^2 q + \alpha q + \beta q_{tt} + \beta_3 q_{ttt}$$
$$\gamma(\, q |^2 q)_t + iR(| q \; ^2)_t q \qquad (12)$$

where $t = (T - Z/v_g)/t_0$ and $z = Z/z_0$ are normalized time and propagation cordinates, with t_0 an arbitrary time, $z_0 = t_0^2/|k''|$, and with v_g and k'' the group velocity and the group-velocity dispersion; $\beta_3 = k'''/(6 \, k'' | t_0)$, with k''' the third-order dispersion; $\gamma = 2/(\omega_0 t_0)$ and $R = -T_R/t_0$ are the NLD and IRS coefficients; α is the excess amplifier gain; and β is the filter depth. Note that TOD, NLD, and IRS scale with t_0 in the same manner.

6 Suppression of Noise Accumulation in Bandwidth Limited Soliton Transmission by Means of Nonlinear Loop Mirrors

M. Matsumoto et al.[28] showed that the amplification and accumulation of noise and other dispersive waves in bandwidth-limited soliton transmission can be suppressed by the use of nonlinear amplifying loop mirrors that selectively amplify solitons. This scheme offers an alternative possibility for achieving stable ultralong-distance optical soliton transmission.

Two methods have recently been proposed to reduce the Gordon-Haus timing jitter of solitons in fibers[29] induced by amplifier noise. One is to modulate pulse trains synchronously in the time domain to retime the pulse position by high-speed optical modulators[30]. The other is to suppress the diffusion of the soliton center frequency by narrow-band filters[31,17]. In the latter method, which has the advantages of being passive and being adaptable to wavelength-division multiplexing, the problem of linear-wave growth caused by the excess gain around the filter center frequency must be settled. Mollenauer et al. recently demonstrated that sliding the center frequency of the filter along the transmission distance is quite effective in eliminating the accumulation of noise. Introduction of nonlinear gains that selectively amplifies solitons have also been proposed to suppress the growth of linear waves[32,33]. M. Matsamoto et al. propose to use nonlinear amplifying loop mirrors (NLM's) periodically inserted in the transmission line as a high-speed nonlinear-gain element and present numerical simulation of stable soliton transmission.

The perturbed nonlinear Schrödinger equation for the distributed model is

$$i\frac{\partial u}{\partial Z}+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2u=$$
$$-\frac{B}{Z_{NLM}}|u|^2u+i\beta\frac{\partial^2 u}{\partial T^2}+$$
$$i\left(\delta+\frac{1}{Z_{NLM}}\ln A\right)u+i\frac{2}{Z_{NLM}}(\ln|u|)u \quad (13)$$

where $\beta(>0)$ is a parameter representing filter strength and δ is the excess gain. Z_{NLM} is an assumed period of insertion of a NLM normalized to the dispersion lenth. Equation (13) reveals that the nonlinear gain coefficient is proportional to $\ln|u|$, which means that linear waves with $|u|<1$ are forced to decay. Taking terms is the right-hand side of Eq. (13) as perturbations and assuming the solution of Eq. (13) in the form $u=\eta\operatorname{sech}h\eta(T+\kappa Z)\exp[-i\kappa T+i(\eta^2-\kappa^2)/2]$. We obtain the following equations describing the evolution of η and κ (Ref. 21):

$$\frac{\partial\eta}{\partial Z}=2\delta'\eta-2\beta\eta\left(\frac{1}{3}\eta^2+\kappa^2\right)+\frac{4}{Z_{NLM}}\eta\ln\eta \quad (14a)$$

$$\frac{\partial\kappa}{\partial Z}=-\frac{4}{3}\beta\kappa\eta^2 \quad (14b)$$

where $\delta'=\delta+(\ln(A+c)/Z_{NLM}$ and $c=\int_{-\infty}^{\infty}\operatorname{sec}h^2x\ln(\operatorname{sec}hx)dx\cong-0.6137$. The effect of the nonlinear gain is determined solely by the spacing between nonlinear loop mirrors Z_{NLM}. Equations (14) have a stationary solution (η_0, $\kappa_0=0$), where η_0 is a solution of $2\delta'-2\beta\eta_0^2/3+(4/Z_{NLM})\ln\eta_0=0$. The solution is stable for $Z_{NLM}>3/(\beta\eta_0^2)$, which means that the insertion of nonlinear loop mirrors may cause instability if the insertion period Z_{NLM} is appropriately small.

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非线性Schrödinger方程在光纤通讯中的应用

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摘 要：综述了非线性Schrödinger方程在光纤通讯系统中的一些应用，主要内容有：1)光纤中N孤子相互作用；2)光纤中的脉冲传播；3)在色散移位光纤中的飞秒孤子-孤子相互作用；4)强耦合非线性参数孤波；5)光纤中有限带宽孤子传播的高阶效应；6)用非线性环形反射镜对有限带宽孤子传输中的噪声积累进行抑制。

关键词：通讯系统；非线性薛定谔方程；光纤；孤波；孤子

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Mannich Reactions of Aromaitic Aldehydes with Aromatic Amines and Aromatic Ketones

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ABSTRACT: Mannich reactions of aromatic aldehydes with aromatic amines amines and aromatic ketones are performed at room temperature or slightly below in the presence of concetrated hydrochloric acid as the catalyst. Forty-one new compounds are synthesized with yields of 18.9% to 92%. The effect of subsrituent in the ring, the amount of hydrochloric acid, reaction temperature and reaction time are systematically examined as well.

KEYWORDS: Mannich bases; Mannich reaction; aromatic aldehydes; aromatic ketones; aromatic amines

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