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Multiple Periodic Solutions with Fixed Period for N – body Type Problems*

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ABSTRACT: Using the equivariant Ljusternik – Schnirelmann theory, the estimates of the lower bound of functional values on collision sets and the estimates of the critical values for the variational functional, we obtain some results in the large concerning multiple geometrically distinct noncollinear periodic solutions of any fixed mininal period for a class of planar N-body type problems.

KEYWORDS: N-body-type problems with fixed minimal period; geometrically distinct noncollinear periodic solutions; equivariant Ljusternik – Schnirelmann theory; symmetry

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The N-body problem is a difficult classical problem in celestial mechanics and differential equations. It is related to the moving law of N point masses, which are governed by the Newton's second law and the universal gravitation law.

$$m_i\ddot{q}_i + \frac{\partial V}{\partial q_i} = 0 \qquad i = 1, \cdots, N \tag{1}$$

where $q_i \in R^k$ (where k is a finite integer greater than or equal to 1) is the position and m_i is the mass of the ith particle, respectively, and the potential V is given by

$$V(q_1, \cdots, q_N) = -\sum_{1 \le i < j \le N} \frac{m_i m_j}{|q_i - q_j|^\alpha}$$

The study of some special noncollision solutions for N-body problems has a long history[1 ~ 41]. For planar 3-body problem, in 1767, Euler [1] discovered three collinear periodic solutions, and in 1773, Lagrange [2] discovered two equilateral periodic solutions.

In recent years, some researchers have applied the variational methods to the study of the existence and multiplicity of periodic solutions for certain classes of N-body type problems. Using the equivariant Ljusternik – Schnirelmann theory, we can obtain some new results in the large concerning with multiple geometrically distinct noncollinear periodic solutions of fixed minimal period for some planer N-body problems.

We set $\Omega = R^2 \setminus \{0\}$ and consider a potential V of the form:

$$V(u) = V(u_1, \cdots, u_N) = \frac{1}{2}\sum_{1 \le i \ne j \le N} V_{ij}(u_i - u_j) \tag{2}$$

where $u_i \in R^2$, $u = (u_1, \cdots, u_N) \in (R^2)^N$ and $V_{ij} \in C^1(\Omega, R)$.

Given $T > 0$, we seek periodic solutions of

$$\left.\begin{aligned} & m_i\ddot{u}_i + \frac{\partial}{\partial u_i}V(u_1, \cdots, u_N) = 0, 1 \le i \le N \\ & u_i(0) = u_i(T), \dot{u}_i(0) = \dot{u}_i(T), 1 \le i \le N \end{aligned}\right\} \tag{PT}$$

Definition 1　If the solution $u = (u_1, \cdots, u_N)$ of (PT) satisfies

(i) $u_i \in C^2(0, T; R^2)$;

(ii) $u_i(t) \ne u_j(t)$, $\forall\, t \in [0, T]$ and $1 \le i \ne j \le N$.

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Then we say that u is the noncollision periodic solution of (PT).

Definition 2 Let u and v be two periodic solutions of (PT). If there are diffeo - morphisms $\varphi: S^1 \to S^1$ and $R: O(2) \to O(2)$ such that $y = R \cdot x \cdot \varphi$, that is u and v belong to the same orbit and have the same period so that there are $\theta \in S^1$ and $R \in O(2)$ such that $u(t) = Rv(t+\theta)$, then we regard u and v to be the same in geometry; otherwise, u and v distinct in geometry.

Theorem 1 Assume V satisfies

(V1) there are $a > 0$ and $\alpha > 0$ such that for any $u_i \in R^2$ and $u_i \neq u_j$

$$V(u_1, \cdots, u_N) = -\frac{a}{2}\sum_{i \neq j}\frac{m_i m_j}{|u_i - u_j|^\alpha}$$

(V2) $\tilde{C}_5 > C_6$, where

$$C_6 = \frac{\alpha + 18}{9(\alpha + 2)} C_5$$

$$C_5 = \left[\left(\frac{\alpha}{2}\right)^{\frac{2}{\alpha+2}} + \left(\frac{\alpha}{2}\right)^{\frac{-\alpha}{\alpha+2}}\right] \cdot C_3^{\frac{\alpha}{\alpha+2}} \cdot C_4^{\frac{2}{\alpha+2}}$$

$$\tilde{C}_5 = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\alpha}\right)\alpha^{\frac{2}{\alpha+2}} \cdot a^{\frac{2}{\alpha+2}} \cdot (2\pi)^{\frac{2\alpha}{\alpha+2}}\left(\sum_{1 \leq i \neq j \leq N} m_i m_j\right) \cdot M^{\frac{-\alpha}{\alpha+2}}$$

$$C_4 = \frac{a}{2}\sum_{1 \leq i \neq j \leq N}\frac{m_i m_j}{|a_i - a_j|^\alpha}$$

$$C_3 = \sum_{i=1}^{N} 2\pi^2 a_i^2 m_i$$

$$a_i > 0, a_i \neq a_j$$

Then for any $T > 0$, (PT) possesses at least $2(N-1)\cdot 2^{N-2} - N$ geometrically distinct noncollinear and noncollision periodic solutions with T as minimal period.

1 Proof of the Main Results

Let $O(2)$ denote the rotational group in R^2 and

$$A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \in O(2), \forall \theta \in [0, 2\pi]$$

Let us introduce the following notations:

$$H = W^{1,2}(R/TZ, R^2)$$

$$H_* = \{u \in H \mid u(t + T/r) = A\left(\frac{2\pi}{r}\right)u(t)$$

$r \geq 2$ is a positive integer$\}$

$$E = \{u = (u_1, \cdots, u_N) \mid u_i - u_j \in H_*, \ i, j = 1, \cdots, N\}$$

$$\Lambda = \{u \in E \mid \sum_{i=1}^{N} m_i u_i(t) \equiv 0, \ u_i(t) \neq u_j(t), \forall t \in R, i \neq j\}$$

$$L = \{u \in E \mid \text{for some } \lambda_i \in R \text{ and } \forall t \in R, \ i = 2, \cdots, N, u_i(t) = \lambda_i u_1(t)\}$$

$$\Lambda_0 = \Lambda - L$$

$$\langle u, v\rangle = \int_0^T \dot{u}\dot{v}\,dt, \ |u|^2 = \int_0^T \dot{u}^2 dt, \forall u, v \in H_*.$$

On Λ_0 we define the following functional:

$$f(u) = \frac{1}{2}\sum_{i=1}^{N} m_i \int_0^T |\dot{u}_i|^2 dt - \int_0^T V(u)dt$$

It is easy to prove the following variational principle:

Lemma 1 (i) $f \in C^1(\Lambda_0, R)$; (ii) $u \in \Lambda_0$ is the critical point of f, then u is a noncollinear and noncollision periodic solution of (PT).

Lemma 2([30]) Let $X = (x_1, \cdots, x_N) \in (R^k)^N$. Then

$$\frac{1}{2}\sum_{1 \leq i \neq j \leq N}\frac{m_i m_j}{|x_i - x_j|^\alpha} \geq C_1 \frac{1}{(\sum_{i=1}^{N} m_i |x_i|^2)^{\alpha/2}},$$

where $C_1 = 2^{-(\alpha+1)/2} \cdot M^{-\alpha/2}(\sum_{1 \leq i \neq j \leq N} m_i m_j)^{(2+\alpha)/2}$, $M = \sum_{i=1}^{N} m_i$.

Lemma 3 Let $K = \inf\{\liminf_{n \to \infty} f(u_n) \mid \{u_n\} \subset \Lambda_0, u_n \xrightarrow{w} u \in \partial\Lambda_0\}$. Then $\forall \varepsilon > 0$, the (P.S.) cndition holds in $f^{K-\varepsilon} = \{u \in \Lambda_0 \mid f(u) \leq K - \varepsilon\}$.

Proof Let $\{u_n\}$ be a (P.S.) sequence at level $c < K$. From $f(u_n) \to c$, we know that $\int_0^T |\dot{u}_n|^2 dt$ is bounded. Thus $\{u_n\}$ is bounded in Λ_0, taking a subsequence if necessary, we can say $u_n \xrightarrow{w} u$. From $c < K$ we have that $u \in \Lambda_0$. Similar to the proof of [19 ~ 28, 42], $f'(u_n) \to 0$ and $u_n \xrightarrow{w} u \in \Lambda_0$ imply $u_n \to u \in \Lambda_0$.

Lemma 4 Assume (V1) holds, then

$$K \geq C_2 T^{\frac{2-\alpha}{2+\alpha}}$$

where K and C_2 are difined in Lemma 3 and Theorem 1 respectively.

Proof By the definition of K, we take a sequence $\{u_n\} \subset \Lambda_0$ such that $u_n \xrightarrow{w} u \in \partial\Lambda_0$ and $f(u_n) \to K$. We are going to give a lower bound on K.

Let S_N denote the group of all the permutations of $\{1, \cdots, N\}$. We set, for $l = 2, \cdots, N$,

$$\partial\Lambda_l = \{u \in E: \exists s \in S_N, \exists t \in [0, T]$$

such that

$$u_{s(1)}(t) = \cdots = u_{s(l)}(t).$$

Let $u \in \partial \Lambda_l$, we can assume, for simplicity, that s is the identity and $t=0$. Using the symmetry property $(u_i - u_j)(t + T/r) = A\left(\frac{2\pi}{r}\right)(u_i - u_j)(t)$, we deduce that

$$f(u) \geqslant g_l(u) + h_{N-l}(u)$$

where

$$g_l(u) = r\Big[\sum_{i=1}^{l}\frac{m_i}{2}\int_0^{T/r}|\dot u_i(t)|^2 dt + \frac{a}{2}\sum_{1\leqslant i\neq j\leqslant l}\int_0^{T/r}\frac{m_i m_j dt}{|u_i(t)-u_j(t)|^\sigma}\Big]$$

$$h_{N-l}(u) = r\Big[\sum_{l+1}^{N}\frac{m_i}{2}\int_0^{T/r}|\dot u_i(t)|^2 dt + \frac{a}{2}\sum_{l+1\leqslant i\neq j\leqslant N}\int_0^{T/r}\frac{m_i m_j dt}{|u_i(t)-u_j(t)|^\sigma}\Big]$$

Using the methods of [23,28,33,40], it is possible to give more explicit estimates about $g_l(u)$ and $h_{N-l}(u)$.

Let

$$M_l = \sum_{i=1}^{l} m_i$$

$$A = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{\alpha}\right)(\alpha a)^{2/(\sigma+2)}\cdot(2\pi r)^{2\sigma/(\sigma+2)}$$

$$\tilde P_l = A\cdot M_l^{-\sigma/(2+\sigma)}\sum_{1\leqslant i\neq j\leqslant N} m_i m_j$$

Then $\quad g_l(u) \geqslant \tilde P_l T^{\frac{2-\sigma}{2+\sigma}}$

Let

$$J_l = \min_{s\in S_N}\frac{\sum_{1\leqslant i\neq j\leqslant l} m_{s(i)} m_{s(j)}}{\left(\sum_{i=1}^{l} m_{s(i)}\right)^{\sigma/(2+\sigma)}}$$

$$P_l = A\cdot J_l$$

Then

$$\inf_{u\in\partial\Lambda_l} g_l(u) \geqslant P_l T^{\frac{2-\sigma}{2+\sigma}}$$

Let

$$M_{N-l} = \sum_{i=l+1}^{N} m_i, \quad B = r^{-2\sigma/(\sigma+2)}\cdot A$$

$$\tilde Q_l = B\cdot M_{N-l}^{-\sigma/(2+\sigma)}\sum_{l+1\leqslant i\neq j\leqslant N} m_i m_j$$

Then $\quad h_{N-l}(u) \geqslant \tilde Q_l T^{\frac{2-\sigma}{2+\sigma}}$

Let

$$K_l = \min_{s\in S_N}\frac{\sum_{l+1\leqslant i\neq j\leqslant N} m_{s(i)} m_{s(j)}}{\left(\sum_{i=l+1}^{N} m_{s(i)}\right)^{\sigma/(2+\sigma)}}$$

$$Q_l = B\cdot K_l$$

Then

$$\inf_{u\in\partial\Lambda_l} h_{N-l}(u) \geqslant Q_l T^{\frac{2-\sigma}{2+\sigma}}$$

$$\inf_{u\in\partial\Lambda_l} f(u) \geqslant (P_l + Q_l) T^{\frac{2-\sigma}{2+\sigma}} \qquad \inf_{u\in\partial\Lambda_0} \geqslant C_2 T^{\frac{2-\sigma}{2+\sigma}}$$

where $C_2 = \min_{2\leqslant l\leqslant N}\{P_l + Q_l\}$.

By the weakly lower semi - continuity of f, we have that $K \geqslant C_2 T^{(2-\sigma)/(2+\sigma)}$.

In the following, we define a $S^1\times O(2)$ invariant set Z and estimate the upper bound of f on Z. Let $\omega = 2\pi/T$ and $a_i \neq a_j$, $a_i > 0$

$$Z_i = \{v(t) = (\bar R a_i(\xi\cos\omega t + \eta\sin\omega t) \mid \xi,\ \eta\in R^2,\ |\xi| = |\eta| = 1, \langle\xi,\eta\rangle = 0\}$$

where positive number $\bar R$ is to be determined. Let

$$Z = Z_1\times\cdots\times Z_N$$

$$Z' = Z - \{\bar R a_1,\cdots,\bar R a_i,\cdots,\bar R a_N\}\times S^1$$

Then Z' is a noncollinear and noncollision $S^1\times O(2)$ invariant set and $Z'\subset\Lambda_0$ and for any $u = (u_1,\cdots,u_N)\in Z'$, there are $\xi_i, \eta_i\in R^2$ such that $|\xi_i| = |\eta_i| = 1$, $\xi_i\cdot\eta_i = 0$ and $u_i(t) = (\bar R a_i)(\xi_i\cos\omega t + \eta_i\sin\omega t)$. Hence $|u_i(t)| = \bar R a_i$ and

$$|u_i(t) - u_j(t)|^2 \geqslant |u_i|^2 + |u_j|^2 - 2|u_i||u_j| = [\bar R|a_i - a_j|]^2$$

Hence (V1) implies

$$-V(u) \leqslant \left(\frac{a}{2}\cdot\sum_{i\neq j}\frac{m_i m_j}{|a_i - a_j|^\sigma}\right)\cdot\bar R^{-\sigma}$$

On the other hand, by $|\dot u_i| = \omega\bar R a_i$, we have

$$\sum_{i=1}^{N} m_i\int_0^T|\dot u_i|^2 dt = \left(\sum_{i=1}^{N}\omega^2 m_i a_i^2\right)T\bar R^2$$

We deduce that, $\forall\,\bar R > 0$

$$f(u) \leqslant C_3 T^{-1}\bar R^2 + C_4 T\bar R^{-\sigma} = h(\bar R)$$

where

$$C_3 = \sum_{i=1}^{N} 2\pi^2 m_i a_i^2; \quad C_4 = \frac{a}{2}\sum_{i\neq j}\frac{m_i m_j}{|a_i - a_j|^\sigma}$$

When $\quad R_0 = \left(\frac{a C_4}{2C_3}\right)^{\frac{1}{\sigma+2}} T^{\frac{2}{\sigma+2}}$

we have that $h'(R_0) = 0$, $h''(R_0) > 0$. Hence $h(\bar R)$ has unique minimum value:

$$h(R_0) = \left[\left(\frac{a}{2}\right)^{\frac{2}{\sigma+2}} + \left(\frac{a}{2}\right)^{\frac{-\sigma}{\sigma+2}}\right]\cdot C_3^{\frac{\sigma}{\sigma+2}} C_4^{\frac{2}{\sigma+2}}\cdot T^{\frac{2-\sigma}{2+\sigma}} = C_5\cdot T^{\frac{2-\sigma}{2+\sigma}}$$

Hence we have

Lemma 5 If we choose positive number $\bar R$ of the set

Z as R_0, then $\quad k = \max_{u\in Z} f(u) \leqslant C_5 \cdot T^{\frac{2-\alpha}{2+\alpha}}$

By [27] we know that if u is a critical point of f in Λ_0, then $\quad \inf_{u\in\Lambda_0} f(u) \geqslant \tilde{C}_5 T^{\frac{2-\alpha}{2+\alpha}}$

We note that each $Z_i\ (i = 1,\cdots,N)$ is diffeomorphic to $T_1 S^1$ which is the unit tangent bundle of S^1. Hence cat $(Z_i) \geqslant 3$. Now $Z = Z_1 \times \cdots \times Z_N$ is diffeomorphic to $T_1 S^1 \otimes \cdots \otimes T_1 S^1$ (N factors), and Z is diffeomorphis to the set $Z_{(n_1,\cdots,n_N)}$ of Coti Zelati([29]), so by [29], we have

Lemma 6

$$\text{cat}\,(Z/S^1 \times O(2)) \geqslant 2\cdot(N-1)\cdot 2^{N-2}$$

$$\text{cat}\,(Z'/S^1 \times O(2)) \geqslant 2\cdot(N-1)\cdot 2^{N-2} - N$$

The following lemma is an equivariant version of Ljusternik – Schnirelmann theory.

Lemma 7　Let X be a Banach space, Λ an open subset of X, and $f \in C^1(\Lambda, R)$. Let G be a compact Lie group, $T(G)$ a linear continuous representation with equivariant distance and M a C^{2-0} submanifold of Λ. Assume M and f both are invariant under $T(G)$, f satisfies the Palais – Smale condition on some closed subset N of M.

Let i be a $T(G)$ – invariant index. Let $c_m = \inf_{i(A)\geqslant m} \sup_{x\in A} f(x)$, $m = 1,2,\cdots$, where $A \subset N$, $A \in \Sigma = \{B \subset N \mid B$ is $T(G)$-invariant and closed in $N\}$, then

(1) when $-\infty < c_m < +\infty$, c_m is a critical value of f;

(2) if $-\infty < c = c_{m+1} = \cdots = c_{m+k} < +\infty$, then $i(K_c) \geqslant k$, where $K_c = \{x \in N \mid f'(x) = 0, f(x) = c\}$;

(3) $c_m \leqslant c_{m+1}$

Lemma 8　If r is an even positive integer and u is a critical point of f in f^k, which has minimal period T/m, $m \neq 1$, then $m \geqslant 3$ and $v(t) = u(t/m)$ satisfies that

$$f(v) \leqslant C_6 T^{\frac{2+\alpha}{2+\alpha}}$$

where $C_6 = (\alpha+18)\cdot[9(\alpha+2)]^{-1}\cdot C_5$

Proof　(i) From the even property for r and $(u_i - u_j)\left(t + \frac{1}{r}\right) = A(2\pi/r)(u_i - u_j)(t)$ we have $(u_i - u_j)\left(t + \frac{1}{2}\right) = -(u_i - u_j)(t)$. $m \geqslant 3$ follows from the symmetry $(u_i - u_j)(t + T/2) = -(u_i - u_j)(t)$ and $u \in f^k \subset \Lambda_0$.

(ii) By the definition of v we have

$$\int_0^T |\dot{v}(t)|^2 dt = \frac{1}{m^2}\int_0^T |\dot{u}(t)|^2 dt$$

$$\int_0^T V(v)dt = \int_0^T V(u)dt$$

By the process of the proof of Lemma 5, we have that

$$f(v) = \frac{1}{2}\int_0^T \sum_{i=1}^N m_i\,|\dot{v}_i|^2 dt - \int_0^T V(v)dt \leqslant$$

$$\frac{1}{9}\cdot\frac{1}{2}\sum_{i=1}^N \int_0^T m_i\,|\dot{u}_i|^2 dt - \int_0^T V(u)dt$$

By $\langle f'(u), u\rangle = 0$, we have

$$\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt - \int_0^T \langle V'(u), u\rangle dt = 0$$

$$\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt = -\alpha\int_0^T V(u)dt$$

$$-\int_0^T V(u)dt = \frac{1}{\alpha}\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt$$

By $u \in f^k$, we have that

$$f(u) = \frac{1}{2}\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt - \int_0^T V(u)dt,$$

$$\left(\frac{1}{2} + \frac{1}{\alpha}\right)\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt \leqslant k,$$

$$\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt \leqslant k\cdot\left(\frac{1}{2} + \frac{1}{\alpha}\right)^{-1},$$

$$f(v) \leqslant \frac{1}{18}\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt - \int_0^T V(u)dt =$$

$$\left(\frac{1}{18} + \frac{1}{\alpha}\right)\int_0^T \sum_{i=1}^N m_i\,|\dot{u}_i|^2 dt \leqslant$$

$$k\left(\frac{1}{18} + \frac{1}{\alpha}\right)\left(\frac{1}{2} + \frac{1}{\alpha}\right)^{-1} =$$

$$k\cdot\frac{\alpha+18}{9(\alpha+2)} \leqslant C_6\cdot T^{\frac{2-\alpha}{2+\alpha}}$$

Now Theorem 1 can be proved by Lemma 2 and Lemmas 3 ~ 8.

Proof　Choose r to be a larger even positive integer so that $C_2 > C_5$ holds. Inequality $C_2 > C_5$ implies that $K > k$. Thus by Lemma 3, the Palais Smale condition holds in f^k. We note that $Z \subset f^k$ by the definition of k.

Since $i_{Z'}(Z') \geqslant 2\cdot(N-1)\cdot 2^{N-2} - N$, we are in position to apply the $S^1 \times O(2)$-equivariant Ljusternik – Schnirelmann theory in $N = f^k$ (see Lemma 7) which yields the existence of $2\cdot(N-1)\cdot 2^{N-2} - N$ critical points $\{u_i \mid i = 1,\cdots,2\cdot(N-1)2^{N-2}\} \subset f^k$ of $f|_M$. If u_i were not of minimal period T, from Lemma 8, it would follow that there are v_i such that $f(v_i) \leqslant C_6 \cdot T^{\frac{2-\alpha}{2+\alpha}}$. Since that for any $v_i \in Z'$ we have $f(v_i) \geqslant \tilde{C}_5 \cdot T^{\frac{2-\alpha}{2+\alpha}}$. So Lemma 4 and estimate $\tilde{C}_5 > C_6$ would then imply a contradiction. Hence all critical points u_i have minimal period T and none of which can be brought into the other by the standard $S^1 \times O(2)$ action on M.

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给定周期的 N 体型问题多个几何不同的周期解

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摘　要：利用牛顿势能函的对称性和等变 Ljusternik – Schnirelmann 理论及变分泛函在碰撞集上的值的下界估计和在临界点集上的值的上、下界估计，获得了关于给定极小周期的平面 N 体型问题多个几何不同的非共线的周期解的大范围存在性的新结果。

关键词：具有给定极小周期的 N 体型问题；几何不同的非共线周期解；等变 Ljusternik – Schnirelmann 理论；对称性

存在性

（责任编辑　张小强）

·下期论文摘要预告·

焦耳热效应和矿物质对煤导电性的影响

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摘　要：主要讨论了焦耳热效应和矿物质对煤导电性质的影响。结果表明，因焦耳热的影响煤体温度升高，使得通过煤体的电流增大，且这种效应随时间的增加，由于散热的影响而减弱，从而导致一定时间后导电性随时间而减弱；煤在脱除方解石、石英等矿物质后电导率增大 1 ~ 2 个数量级，煤的堆密度增大及煤粒度减小时其电导率将增大。

关键词：煤；导电性；焦耳热；矿物质；堆密度