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不改变系统能观性的状态反馈的存在性*

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摘　要:现代控制论对能控、能观但不稳定的系统,可用一个线性状态反馈使其变得稳定,并保持系统的能控性不变,但对于这种反馈是否也能保持系统的能观性不变则没有讨论,这是一个理论上的缺憾。这里就一类定常离散系统对这个问题进行了研究,由矩阵的特征根及矩阵的秩与线性定常离散系统的稳定性及能观性的关系,用构造的方法证明了不改变系统能观性的状态反馈的存在性,并给出了构造这种反馈的方法。

关键词:能控;能观;稳定;状态反馈
中图分类号:TP13　　**文献标识码**:A

现代控制理论告诉人们对一个能控、能观但不稳定的系统,可给一个线性状态反馈使它变得稳定,并且不改变该系统的能控性。但这样一个使系统稳定性发生改变的状态反馈是否能使系统的能控性、能观性都不变呢?文中研究了单输入、单输出线性定常离散系统的情形,给出了肯定的回答。

1　预备定理

定理 1　非奇异变换不改变系统的能控性、能观性,系统的特征值也不变[1-2]。

定理 2　n 阶行列式

$$D=\begin{vmatrix}0&\cdots&\cdots&0&s_{m+1}&\cdots&s_n\\ rs_n&\ddots&&&\ddots&\ddots&\vdots\\ \vdots&\ddots&\ddots&&&\ddots&s_{m+1}\\ rs_{m+1}&&\ddots&\ddots&&&0\\ 0&\ddots&&\ddots&\ddots&&\vdots\\ \vdots&\ddots&\ddots&&\ddots&\ddots&\vdots\\ 0&\cdots&0&rs_{m+1}&\cdots&rs_n&0\end{vmatrix}=(-1)^{m(n-1)}r^mP(r),$$

其中 $s_1=\cdots=s_m=0,s_{m+1}\neq 0,s_{m+2},\cdots,s_n(1\leqslant m<n)$ 为常数。而 $P(r)$ 是关于 r 的多项式,其常数项为 s_{m+1}^n。

证　首先在行列式左边 m 列的每列提取因子 r,再通过列的两两交换将这 m 列交换到右边得

$$D=(-1)^{m(n-1)}r^m\begin{vmatrix}s_{m+1}&\cdots&\cdots&s_n&0&\cdots&0\\ 0&\ddots&&&\ddots&\ddots&\vdots\\ \vdots&\ddots&\ddots&&&\ddots&0\\ 0&&\ddots&\ddots&&&s_n\\ rs_n&\ddots&&\ddots&\ddots&&\vdots\\ \vdots&\ddots&\ddots&&\ddots&\ddots&\vdots\\ rs_{m+1}&\cdots&rs_n&0&\cdots&0&s_{m+1}\end{vmatrix}$$

要证明定理 2,只须证明上式中的行列式因子是关于 r 的多项式,且常数项是 s_{m+1}^n。第 1 个结论是显然的,根据此行列式的形式,由行列式按一行展开的定理[3],即知该多项式的常数项是 s_{m+1}^n。证毕。

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定理 3　设单输入单输出的线性定常系统

$$\begin{cases}X_{k+1}=AX_k+BU_k\\ Y_k=CX_k\end{cases}\tag{1}$$

其中 A,B,C 分别为 $n\times n,n\times 1,1\times n$ 阶常系数矩阵 $X_k=(x_1^{(k)},x_2^{(k)},\cdots,x_n^{(k)})^{\mathrm T}$,为 n 维动态向量。系统(1)是能控、能观但不稳定的,则一定存在线性状态反馈 $U_k=V_k-KX_k$,使得反馈系统

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$$\begin{cases} X_{k+1} = (A - BK)X_k + BV_k \\ Y_k = CX_k \end{cases} \tag{2}$$

变得稳定,并且保持系统的能控、能观性,其中 K 为 $1 \times n$ 阶反馈矩阵。

证　非奇异的初等变换 $\tilde{X}_k = TX_k$(其中 $T = (P_1 \vdots P_1A \vdots \cdots \vdots P_1A^{n-1})^{\mathrm{T}}$,而 $P_1 = (0,\cdots,0,1)(B \vdots AB \vdots \cdots \vdots A^{n-1}B)^{-1}$)可将系统(1)化为能控标准形[4-5]

$$\begin{cases} \tilde{X}_{k+1} = \tilde{A}\tilde{X}_k + \tilde{B}U_k \\ Y_k = \tilde{C}\tilde{X}_k \end{cases} \tag{3}$$

其中 $\tilde{A} = TAT^{-1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_2 & -a_1 \end{bmatrix}$,

而 $a_1, a_2, \cdots, a_n$ 均为常数,其中 $\tilde{B} = TB = (0,\cdots,0,1)^{\mathrm{T}}$,$\tilde{C} = CT^{-1} = (\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n)$,其中 $\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n$ 是不全为 0 的常数。

由定理 1 可知,系统(3)与系统(1)的能控性和能观性是一致的,稳定性也是一致的。其次通过反馈 $U_k = V_k - H\tilde{X}_k$(其中 $H = (h_1, h_2, \cdots, h_n)$是反馈矩阵),又可以将系统(3)化为

$$\begin{cases} \tilde{X}_{k+1} = (\tilde{A} - \tilde{B}H)\tilde{X}_k + \tilde{B}V_k \\ Y_k = \tilde{C}\tilde{X}_k \end{cases} \tag{4}$$

这里

$$\tilde{A} - \tilde{B}H = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -(a_n + h_1) & -(a_{n-1} + h_2) & -(a_{n-2} + h_3) & \cdots & -(a_2 + h_{n-1}) & -(a_1 + h_n) \end{bmatrix}$$

显然系统(4)是能控的。根据离散系统稳定和能观的条件,要使系统(4)稳定且能观,一方面要使此系统的特征多项式

$$\det[\lambda I - (\tilde{A} - \tilde{B}H)] = \begin{vmatrix} \lambda & -1 & \cdots & 0 & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda & -1 \\ a_n + h_1 & a_{n-1} + h_2 & \cdots & a_2 + h_{n-1} & \lambda \end{vmatrix} =$$

$$\lambda^n + (a_1 + h_n)\lambda^{n-1} + (a_2 + h_{n-1})\lambda^{n-2} + \cdots + (a_{n-1} + h_2)\lambda + (a_n + h_1)$$

的特征根都在单位圆内[6],即 $|\lambda| < 1$;另一方面须使此系统的能观性矩阵

$$\boldsymbol{\Gamma} = \begin{bmatrix} \tilde{C} \\ \tilde{C}(\tilde{A} - \tilde{B}H) \\ \tilde{C}(\tilde{A} - \tilde{B}H)^2 \\ \vdots \\ \tilde{C}(\tilde{A} - \tilde{B}H)^{n-1} \end{bmatrix}$$

满秩[4]。为此笔者取 $h_1 = -(r + a_n)$,$h_2 = -a_{n-1}$,$\cdots$,$h_{n-1} = -a_2$,$h_n = -a_1$,这时特征多项式为

$$\det[\lambda I - (\tilde{A} - \tilde{B}H)] = \lambda^n - r \tag{5}$$

下面分两种情况来讨论:

1) 当 $\tilde{c}_1 \neq 0$,令 $r = 0$,由式(5)知 $\lambda = 0$ 是特征多项式的 n 重根。显然有 $|\lambda| < 1$,这时 $\det \Gamma =$

$$\begin{vmatrix} \tilde{c}_1 & \tilde{c}_2 & \cdots & \tilde{c}_n \\ 0 & \tilde{c}_1 & \cdots & \tilde{c}_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{c}_1 \end{vmatrix} = \tilde{c}_1^n \neq 0$$,即 Γ 满秩。

2) 当 $\tilde{c}_1 = \cdots = \tilde{c}_m = 0$,而 $\tilde{c}_{m+1} \neq 0 (1 \leqslant m < n)$时

$$(\tilde{A} - \tilde{B}H) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ r & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$(\tilde{A} - \tilde{B}H)^2 = \begin{bmatrix} 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ r & 0 & 0 & \cdots & 0 \\ 0 & r & 0 & \cdots & 0 \end{bmatrix},$$

$$\cdots, (\tilde{A} - \tilde{B}H)^{n-1} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \\ r & 0 & 0 & \cdots & 0 & 0 \\ 0 & r & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & r & 0 \end{bmatrix}$$

于是 $\boldsymbol{\Gamma}=\begin{bmatrix} 0 & \cdots & \cdots & \cdots & \tilde{c}_{m+1} & \cdots & \tilde{c}_n \\ r\tilde{c}_n & \ddots & & & & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & & \tilde{c}_{m+1} \\ r\tilde{c}_{m+1} & & \ddots & \ddots & & & \vdots \\ 0 & \ddots & & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & r\tilde{c}_{m+1} & \cdots & r\tilde{c}_n & 0 \end{bmatrix}$

由定理 2 知,$\det\Gamma=(-1)^{m(n-1)}r^mP(r)$,其中 $P(r)$ 是常数项为 $\tilde{c}_{m+1}^n$ 的多项式。而 $\tilde{c}_{m+1}\neq 0$,所以 $P(r)$ 是非零多项式。

由于多项式的零点只有有限个,因此总存在满足 $0<|r|<1$ 的 $\bar{r}$ 使 $\bar{r}^mP(\bar{r})\neq 0$。即当 r 取值 $\bar{r}$ 时,Γ 满秩,且这时特征多项式 $\lambda^n-\bar{r}$ 的特征根都在单位圆内。

综上所述,根据 $\tilde{C}$ 的不同情况,取相应的反馈矩阵 H,可使系统(4)能控、能观且稳定。注意,现将 $\tilde{X}_k=TX_k$ 代入(4)得

$$\begin{cases} X_{k+1}=(A-BHT)X_+BV_k \\ Y_k=CX_k \end{cases} \tag{6}$$

由定理 1 知,系统(6)也是能控、能观且稳定的。再设 $K=HT$,则系统(6)就是系统(2),这说明 $U_k=V_k-KX_k$ 即是定理所要求的线性状态反馈。

3 例子

设系统

$$\begin{cases} X_{k+1}=AX_k+BU_k \\ Y_k=CX_k \end{cases} \tag{7}$$

其中 $A=\begin{bmatrix}1&1&0&0\\0&1&1&0\\0&0&1&1\\1&0&0&1\end{bmatrix}$, $B=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $C=(1,0,0,1)$。容易验证此系统能控、能观但不稳定。现在求 $\tilde{A},\tilde{B},\tilde{C}$,因为 $P_1=(0,0,0,1)(B\vdots AB\vdots A^2B\vdots A^3B)^{-1}=(0,1,0,0)$,所以 $T=\begin{bmatrix}P_1\\P_1A\\P_1A^2\\P_1A^3\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&1&1&0\\0&1&2&1\\1&1&3&3\end{bmatrix}$,因此,$\tilde{A}=TAT^{-1}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&4&-6&4\end{bmatrix}$, $\tilde{C}=CT^{-1}=(0,1,-2,1)$,

这里 $a_1=-4,a_2=6,a_3=-4,a_4=0,\tilde{c}_1=0,\tilde{c}_2=1,\tilde{c}_3=-2,\tilde{c}_4=1$,于是取 $H=(-r-a_4,-a_3,-a_2,-a_1)=(-r,4,-6,4)$,则 $\boldsymbol{\Gamma}=\begin{bmatrix}\tilde{C}\\\tilde{C}(\tilde{A}-\tilde{B}H)\\\tilde{C}(\tilde{A}-\tilde{B}H)^2\\\tilde{C}(\tilde{A}-\tilde{B}H)^3\end{bmatrix}=\begin{bmatrix}0&1&-2&1\\r&0&1&-2\\-2r&r&0&1\\r&-2r&r&0\end{bmatrix}$,当取 $r=-1/2$ 时,$H=(1/2,4,-6,4)$,因此 $\det\boldsymbol{\Gamma}=\frac{1}{2}\begin{vmatrix}0&1&-2&1\\1&0&1&-2\\-2&-1/2&0&1\\1&1&-1/2&0\end{vmatrix}=\frac{3}{8}\neq 0$,于是所求的反馈矩阵为 $K=HT=(4,5/2,4,6)$,则所求的状态反馈为 $U_k=V_k-(4,5/2,4,6)X_k$,代入(7)得

$$\begin{cases} X_{k+1}=\begin{bmatrix}-3&-3/2&-4&-6\\0&1&1&0\\0&0&1&1\\1&0&0&1\end{bmatrix}X_k+\begin{bmatrix}1\\0\\0\\0\end{bmatrix}V_k \\ Y_k=(1,0,0,1)X_k \end{cases} \tag{8}$$

不难验证系统(8)能控、能观且稳定。

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Application and Progress in Cancer Analysis by FTIR Spectroscopy

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Abstract: The research progress and the application of FTIR spectroscopy on diagnosing cancer are reviewed. The cancer is one of the tallest diseases of the rate of human beings. At present, its diagnosis primarily depends on the pathology one. The process of pathology diagnosis is bother and often suffers the influence of the artificial factors. Along with the rapid development of FTIR spectroscopy and its correlative technique, the FTIR spectroscopy has been applied not only to studying the structure of big biologic molecules like protein or nucleic acid, but also to studying more and more complicated system like cells and tissues. The research expresses that FTIR spectroscopy can open out the characteristics of tissues of cancer from the molecular level, and it is very possible that FTIR will be a quantitative instrument of diagnosing cancer.

Key words: FTIR spectroscopy; cancer; analysis; progress

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Existence of a State Feedback of not Changing the Observability of the System

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Abstract: By modern cybernetics, we can find out a state feedback for a linear time system, which does not change the controllability and stability of the system. But whether a state feedback for linear time system can be found out, which does ont change observability of the system or not, so far hasnt been discussed. The problem is studied according to the steady discrete time linear system, and a state feedback is found out, which does ont change the controllability, observability and stability, by relation of stability, observability and characteristic root, rank of matrix, of steady discrete time linear system; the method of how to construct the state feedback is given.

Key words: controllability; observability; stability; state feedback

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