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$\tilde{\rho}$混合序列的 Marcinkiewicz 强大数律与完全收敛性*

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摘 要:讨论了$\tilde{\rho}$混合序的 Marcinkiewicz 强大数律与完全收敛性,所得的结果改进文献[3-4]相应的结果,并得到了完全收敛速度与矩条件之间的等价关系.

关键词:$\tilde{\rho}$混合序列;Marcinkiewicz 强大数律;完全收敛性

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设$\{X_n,n\geqslant 1\}$是概率空间(Ω,B,P)上的随机变量序列,$F_s=\sigma(X_i,i\in S\subset N)$为$\sigma$-域,在$B$中给定$\sigma$-域$F,R$,令

$$\rho(F,R)=\sup\{|corr(X,Y)|,X\in L_2(F),\ Y\in L_2(R)\},$$

其中$\mathrm{corr}(X,Y)=\dfrac{EXY-EXEY}{\sqrt{\mathrm{Var}X\mathrm{Var}Y}}$为相关系数. 下面引入如下的相依系数:对$k\geqslant 0$,令

$\tilde{\rho}(k)=\sup\{\rho(F_s,R_T)$,有限子集$S,T\subset N$,且$\mathrm{dist}(S,T)\geqslant k\}$,显然,$0\leqslant\tilde{\rho}(k+1)\leqslant\tilde{\rho}(k)\leqslant 1$,且$\tilde{\rho}(0)=1$.

定义1 若存在$k\in N$,使$\tilde{\rho}(k)<1$,则称$\{X_i,i\in N\}$为$\tilde{\rho}$混合序列.

对于$\tilde{\rho}$混合序列的讨论,可以参考[1-7]中的有关内容. 笔者讨论了$\tilde{\rho}$混合序列的 Marcinkiewicz 强大数律与完全收敛性,所得的结果(见定理1和推论2)改进了[3]及[4]中相应的结果,并得到了完全收敛速度与矩条件之间的等价关系(见定理2).

1 定理及证明

本文中"$\ll$"表示"O",C为正常数,且在不同的地方可为不同的值.

引理1[6] 设$\{X_i,i\geqslant 1\}$为$\tilde{\rho}$混合序列,$EX_i=0$,$E|X_i|^p<\infty$,$p\geqslant 2$. 则存在一个仅与$\tilde{\rho}$和p有关的正常数C,有

$$E\max_{1\leqslant k\leqslant n}|\sum_{i=1}^{k}X_i|^p\leqslant C\{\sum_{i=1}^{k}E|X_i|^p+(\sum_{i=1}^{k}EX_i^2)^{p/2}\}.$$

定理1 设$\{X,X_i,i\geqslant 1\}$为同分布$\tilde{\rho}$混合序列,$E|X|^p<\infty$,$S_n=\sum_{i=1}^{n}X_i$,$\alpha>\dfrac{1}{2}$,$p\alpha\geqslant 1$,若$\alpha\leqslant 1$时,令$EX=0$. 则

$$\forall\varepsilon>0,\sum_{n=1}^{\infty}n^{p\alpha-2}P(\max_{1\leqslant j\leqslant n}|S_j|>\varepsilon n^{\alpha}<\infty. \quad (1)$$

证明 $\forall i\geqslant 1$,令$X_i^{(n)}=X_iI(|X_i|\leqslant n^{\alpha})$,$S_j^{(n)}=\sum_{i=1}^{j}(X_i^{(n)}-EX_i^{(n)})$. $\forall_{\varepsilon}>0$,则有

$$P(\max_{1\leqslant j\leqslant n}|S_j|>\varepsilon n^{\alpha})\leqslant P(\max_{1\leqslant j\leqslant n}|X_j|>n^{\alpha})+P(\max_{1\leqslant j\leqslant n}|S_j^{(n)}|>\varepsilon n^{\alpha}-\max_{1\leqslant j\leqslant n}|\sum_{i=1}^{j}EX^{(n)}_i|). \quad (2)$$

下证,当$n\to\infty$时,

$$n^{-\alpha}\max_{1\leqslant j\leqslant n}|\sum_{i=1}^{j}EX_i^{(n)}|\to 0. \quad (3)$$

事实上,1)当$\alpha>1$,$p\geqslant 1$时,

$$n^{-\alpha}\max_{1\leqslant j\leqslant n}|\sum_{i=1}^{j}EX_i^{(n)}|\leqslant n^{-\alpha}\sum_{i=1}^{n}E|X_i|I(|X_i|\leqslant n^{\alpha}\leqslant n^{1-\alpha}E|X|I(|X|\leqslant n^{\alpha})\to 0. \quad (4)$$

2)当$\alpha>1$,$p<1$时,由式(4)知

$$n^{-\alpha}\max_{1\leqslant j\leqslant n}|\sum_{i=1}^{j}EX_i^{(n)}|\leqslant n^{1-\alpha}\sum_{k=1}^{n}E|X|I(k-1<$$

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$$|X|^{\frac{1}{\alpha}} \leqslant k).$$

由 $\alpha p \geqslant 1$ 得

$$\sum_{k=1}^{\infty} k^{1-\alpha} E|X| I(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) \leqslant \sum_{k=1}^{\infty} k^{1-\alpha p} E|X|^p I \cdot (k-1<|X|^{\frac{1}{\alpha}} \leqslant k) \leqslant \sum_{k=1}^{\infty} E|X|^p I(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) = E|X|^p < \infty,$$

所以由 Kronecker 引理得 $n^{1-\alpha} \sum_{k=1}^{n} E|X| I(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) \to 0, n \to \infty$,

故

$$n^{-\alpha} \max_{1 \leqslant j \leqslant n} |\sum_{i=1}^{j} EX_i^{(n)}| \to 0, n \to \infty. \qquad (5)$$

3)当 $\frac{1}{2} \leqslant \alpha \leqslant 1$ 时,由 $p\alpha \geqslant 1$ 及 $EX=0$ 得

$$n^{-\alpha} \max_{1 \leqslant j \leqslant n} |\sum_{i=1}^{j} EX_i^{(n)}| \leqslant n^{-\alpha} \sum_{i=1}^{n} E|X_i| I(|X_i|>n^\alpha) \leqslant n^{1-\alpha p} E|X|^p I(|X| \leqslant n^\alpha) \to 0. \qquad (6)$$

由式(4)、(5)及(6)知式(3)成立. 因此当 n 充分大时,由式(2)及(3)知 $P(\max_{1 \leqslant j \leqslant n} |S_j| > \varepsilon n^\alpha) \leqslant \sum_{j=1}^{n} P(|X_j| > n^\alpha) + P(\max_{1 \leqslant j \leqslant n} |S_j^{(n)}| > \frac{\varepsilon}{2} n^\alpha)$. 因此只需证

$$I = \sum_{n=1}^{\infty} n^{\alpha p-2} \sum_{j=1}^{n} P(|X_j| > n^\alpha) < \infty, \qquad (7)$$

$$II = \sum_{n=1}^{\infty} n^{\alpha p-2} P(\max_{1 \leqslant j \leqslant n} |S_j^{(n)}| > \frac{\varepsilon}{2} n^\alpha) < \infty, \qquad (8)$$

由 $E|X|^p < \infty$ 知

$$I = \sum_{n=1}^{\infty} n^{\alpha p-1} P(|X_1| > n^\alpha) \ll E|X|^p < \infty. \qquad (9)$$

由引理 1 知

$$II \ll \sum_{n=1}^{\infty} n^{\alpha p-2-\alpha p} E \max_{1 \leqslant j \leqslant n} |S_j^{(n)}|^q \ll \sum_{n=1}^{\infty} n^{\alpha(p-q)-2} \{\sum_{j=1}^{n} E \cdot |X_j^{(n)}|^q + (\sum_{j=1}^{n} E|X_j^{(n)}|^2)^{q/2} = II_1 + II_2. \qquad (10)$$

当 $0<p<2$ 时,令 $q=2$;当 $p \geqslant 2$ 时,令 $q=(\alpha p-1)/(\alpha-\frac{1}{2})+p$,则

$$II_1 = \sum_{n=1}^{\infty} n^{\alpha(p-q)-1} E|X|^q I(|X| \leqslant n^\alpha) = \sum_{n=1}^{\infty} n^{\alpha(p-q)-1} \cdot \sum_{k=1}^{n} E|X|^q I(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) = \sum_{k=1}^{\infty} \sum_{n=k}^{\infty} n^{\alpha(p-q)-1} E|X|^q I(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) \ll \sum_{k=1}^{\infty} k^{\alpha p} P(k-1<|X|^{\frac{1}{\alpha}} \leqslant k) \ll E|X|^p < \infty. \qquad (11)$$

当 $0<p<2$ 时,令 $q=2$;则 $II_2 = II_1 < \infty$,当 $p \geqslant 2$ 时,

$$II_2 \ll \sum_{n=1}^{\infty} n^{\alpha(p-q)-2+\frac{q}{2}} = \sum_{n=1}^{\infty} n^{\alpha p-2-q(\alpha-\frac{1}{2})} = \sum_{n=1}^{\infty} n^{-1-p(p-\frac{1}{2})} < \infty. \qquad (12)$$

将式(11),(12)代入(10)得 $II<\infty$,至此定理 1 的证明完毕.

推论 1 由定理 1 可推得[4]中的定理 2.

推论 2 由定理 1 可推得 $\tilde{\rho}$ 混合序列情形的 Marcinkiewicz 强大数律. 即在定理 1 中,令 $p\alpha=1$,则由式(1)得 $\lim_{n \to \infty} \frac{S_n}{n^1/p} = 0$ a. s.

推论 2 的证明 由式(1)及 $p\alpha=1$ 知,$\forall \varepsilon>0$ 有 $\sum_{n=1}^{\infty} n^{-1} P(\max_{1 \leqslant j \leqslant n} |S_j| > \varepsilon) < \infty$. 则 $\sum_{k=1}^{\infty} \sum_{n=2^k}^{2^{k+1}-1} (2^{k+1}-1)^{-1} \cdot P(\max_{1 \leqslant j \leqslant 2^k} |S_j| > \varepsilon) < \infty$.

则 $\sum_{k=0}^{\infty} P(\max_{1 \leqslant j \leqslant 2^k} |S_j| > \varepsilon) < \infty$. 所以 $\max_{1 \leqslant j \leqslant 2^k} |S_j| \to 0$ a. s.

$\forall$ 正整数 n,存在非负整数 k_0,使 $2^{k_0} \leqslant n < 2^{k_0+1}$ · $|S_n| \leqslant \max_{1 \leqslant j \leqslant 2^{k_0+1}} |S_j| \to 0$ *a. s.* 因此 $\lim_{n \to \infty} \frac{S_n}{n^{1}_{/p}} = 0$ a. s.

定理 2 在定理 1 中式(1)与 $E|X|^p < \infty$ 等价.

定理 2 的证明 由定理 1 知,要证定理 2 只需证式(1)可推得 $E|X|^p < \infty$ 即可. 而由(1)式可推得 $E|X|^p < \infty$ 的证明类似于[2]中定理 2(*ii*)⇒(*i*)的证明.

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Moment Inequality and Complete Convergence of $\tilde{\rho}$ Mixing Sequences

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Abstract: Marcinkiewicz strong laws and complete convergences of $\tilde{\rho}$ mixing sequences are investigated. The obtained results improve the relevant results obtained before. And the equivelent relationship between rates of complete convergence and moment condition is obtained.

Key words: $\tilde{\rho}$ mixing sequences; Marcinkiewicz strong laws; complete convergence

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Application of Bispectrum Estimation to Analyzing Pulse Signals

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Abstract: According to non-Gaussianity and randomness of the Pulse Signals, the bispectrum estimation is used to analyze the signals for the purpose of extracting the unusual information of the signals of the drug abusers and educing the judgment of how to distinguish drug abusers from healthy persons . The phase of bispectrum carries the important feature information in signal processing. The feature vector of the average phase information based on the indirect nonparametric bispectrum estimation is extracted and used to analyze the Pulse Signals of 15 heroin addicts and 15 healthy persons. It is found that the average phase p_h of heroin addicts on a specified frequency region is generally lower than that of healthy persons. Using the average phase p_h as characteristic parameter, a critical parameter is determined that is used to classify heroin addicts and healthy persons. Thus, all of the 15 healthy addicts are identified. Only one heroin person is misjudged. Experiment results of classification show that the method is feasible and effective.

Key words: bispectrum estimation; average phase; drug abuser; pulse signal

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