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麦克斯韦方程组的另一推导方法*

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摘　要:众所周知,宏观电动力学的理论基础——麦克斯韦方程组是建立在库仑定律、毕奥 - 沙伐尔定律、法拉第电磁感应定律和感生电场、位移电流假设基础上的。在电荷、电流分布随着时间的变化而变化时,如果假定库仑定律和毕奥——沙伐尔定律仍然成立,只需结合法拉第电磁感应定律和感生电场的假设,不需要位移电流假设,用矢量分析,就可推导出麦克斯韦方程组。

关键词:麦克斯韦方程组;磁场;电场;矢量分析
中图分类号:O442　　　**文献标识码**:A

我们知道,宏观电动力学的理论基础麦克斯韦方程组,是建立在库仑定律、毕奥 - 沙伐尔定律,法拉第电磁感应定律和感生电场、位移电流假设基础之上的。在所有涉及到麦克斯韦方程组的教学参考书中[1-4],总是以这样的方式进行推导或讨论的。

首先根据库仑定律和电场强度的定义式,得到电场的表示式[5],

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho(\vec{r}')}{R_3}\vec{R}\,\mathrm{d}v' \tag{1}$$

式中,$\vec{R} = \vec{r} - \vec{r}'$(如图 1 所示)。利用矢量分析,电场的

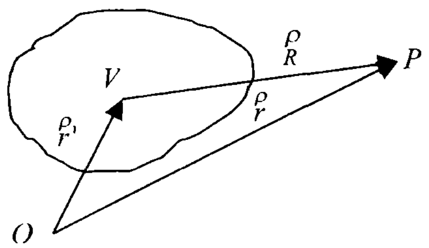

图 1　源点与场点的标记

基本规律:

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \tag{2}$$

$$\nabla \times \vec{E} = 0 \tag{3}$$

由毕奥 - 沙伐尔定律[5]

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi\varepsilon_0}\int_v \frac{\vec{J}(\vec{r}') \times \vec{R}}{R^3}\mathrm{d}v' \tag{4}$$

同样得到恒定磁场的基本规律

$$\nabla \times \vec{B} = \mu_o \vec{J} \tag{5}$$

$$\nabla \cdot \vec{B} = 0 \tag{6}$$

再由法拉第电磁感应定律

$$\varepsilon = -\frac{\mathrm{d}}{\mathrm{d}t}\int_s \vec{B} \cdot \mathrm{d}\vec{s}$$

即

$$\oint_L \vec{E} \cdot \mathrm{d}\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{s} \tag{7}$$

式中的 L 是曲面 S 的边界线,利用斯托克斯公式得感生电场遵从规律

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8}$$

这说明变化的磁场要产生电场,麦克斯韦对该结论进行分析,提出位移电流的假设,即变化的电场与自由电流一样按照相同的规律产生磁场,相应的数学表示式为:

$$\nabla \times \vec{B} = \mu_o \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{9}$$

最后将式(2)、式(6)、式(8)、式(9) 作推广性假设,这样就得到麦克斯韦方程组。

我们所要指出的是,在电荷电流分布随着时间的变化而变化时,如果假定库仑定律和毕奥 - 沙伐尔定律仍然成立,其数学表达式为:

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$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho(\vec{r}',t)}{R^3}\vec{R}\mathrm{d}v' \tag{10}$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int_v \frac{\vec{J}(\vec{r}',t)\times\vec{R}}{R^3}\mathrm{d}v' \tag{11}$$

这样只需结合法拉第电磁感应定律和感生电场的假设,而不需要位移电流假设,同样也能够推导出麦克斯韦方程组。

首先要说明的是,前面推导麦克斯韦方程组时,是在电荷、电流分布附近的区域进行的,而在这个区域电场具有纵向形式,磁场具有横向形式,所以,我们对式(10)、式(11)的假设是可行的,下面我们推导麦克斯韦方程组的过程如下,因为

$$\frac{\vec{R}}{R^3} = -\nabla\frac{1}{R} = \nabla'\frac{1}{R} \tag{12}$$

其中

$$\nabla = \vec{e}_x\frac{\partial}{\partial x} + \vec{e}_y\frac{\partial}{\partial y} + \vec{e}_z\frac{\partial}{\partial z} \tag{13}$$

$$\nabla' = \vec{e}_x\frac{\partial}{\partial x'} + \vec{e}_y\frac{\partial}{\partial y'} + \vec{e}_z\frac{\partial}{\partial z'} \tag{14}$$

故式(11)可写成

$$\boldsymbol{B}(\boldsymbol{r},t) = -\frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r}',t)\times\nabla\frac{1}{R}\mathrm{d}v' = \frac{\mu_0}{4\pi}\int_v \nabla\times\left[\frac{\vec{J}(\vec{r}',t)}{R}\right]\mathrm{d}v' = \nabla\times\boldsymbol{A}(\boldsymbol{r},t) \tag{15}$$

式(15)中的 $\boldsymbol{A}(\boldsymbol{r},t)$ 为

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int_v \frac{\vec{J}(\vec{r}',t)}{R}\mathrm{d}v' \tag{16}$$

所以有

$$\nabla\cdot\vec{B}(\vec{r},t) = \nabla\cdot(\nabla\times\vec{A}) = 0 \tag{17}$$

即磁场是无源场,这就是麦克斯韦方程组之一。而磁场的旋度为:

$$\nabla\times\vec{B} = \nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} \tag{18}$$

先计算 $\nabla\cdot\vec{A}$,由式(14)有

$$\nabla\cdot\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r}',t)\cdot\nabla\frac{1}{R}\mathrm{d}v' = -\frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r}',t)\cdot\nabla'\frac{1}{R}\mathrm{d}v' = -\frac{\mu_0}{4\pi}\int_v \nabla'\cdot\left[\frac{\vec{J}(\vec{r}',t)}{R}\right]\cdot\mathrm{d}v' + \frac{\mu_0}{4\pi}\int_v \frac{\nabla'\cdot\vec{J}(\vec{r}',t)}{R}\mathrm{d}v' = -\frac{\mu_0}{4\pi}\oint_s \frac{\vec{J}(\vec{r}',t)}{R}\cdot\mathrm{d}\vec{s}' + \frac{\mu_0}{4\pi}\int_v \frac{\nabla'\cdot\vec{J}(\vec{r}',t)}{R}\mathrm{d}v' \tag{19}$$

在上式第一项中的 S 是电荷电流分布区域 V 的界面,而在界面 S 上,电流只能沿着界面流动,没有电流通过界面,即在界面 S 上有 $\vec{J}(\vec{r}',t)\cdot\hat{n} = 0$ 或 $\vec{J}(\vec{r}',t) = 0$,因此第一项等于零。把电流连续性方程

$$\nabla\cdot\vec{J}(\vec{r}',t) + \frac{\partial\rho(\vec{r}',t)}{\partial t} = 0 \tag{20}$$

代入式(19)中的第二项,得

$$\nabla\cdot\vec{A} = -\frac{\mu_0}{4\pi}\int_v \frac{1}{R}\frac{\partial\rho(\vec{r}',t)}{\partial t}\mathrm{d}v' = -\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_v \frac{\rho(\vec{r}',t)}{R}\mathrm{d}v'$$

则

$$\nabla(\nabla\cdot\vec{A}) = -\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\nabla\int_v \frac{\rho(\vec{r}',t)}{R}\mathrm{d}v' = \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_v \frac{\rho(\vec{r}',t)}{R^3}\vec{R}\mathrm{d}v'$$

利用(10)式有

$$\nabla(\nabla\cdot\vec{A}) = \mu_0\varepsilon_0\frac{\partial\vec{E}(\vec{r},t)}{\partial t} \tag{21}$$

再计算 $\nabla^2 A(\vec{r},t)$,由式(16)得

$$\nabla^2\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r}',t)\nabla^2\frac{1}{R}\mathrm{d}v' \tag{22}$$

而

$$\nabla^2\frac{1}{R} = -4\pi\delta(\vec{R}) \tag{23}$$

$$\int_v \vec{f}(\vec{r}',t)\delta(\vec{R})\mathrm{d}v' = \vec{f}(\vec{r},t) \tag{24}$$

所以,有

$$\nabla^2\vec{A} = -\mu_0\int_v \vec{J}(\vec{r}',t)\delta(\vec{R})\mathrm{d}v' = -\mu_0\vec{J}(\vec{r},t) \tag{25}$$

将式(21)和式(25)代入式(18)的

$$\nabla\times\vec{B}(\vec{r},t) = \mu_0\vec{J}(\vec{r},t) + \mu_0\varepsilon_0\frac{\partial\vec{E}(\vec{r},t)}{\partial t} \tag{26}$$

即变化的电场与自由电流一样按同样的规律产生磁场。这样我们不需要作位移电流假设,也能得到麦克斯韦方程组之二。

对式(10)取散度,有

$$\nabla\cdot\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int_v \rho(\vec{r}',t)\nabla\cdot\frac{\vec{R}}{R^3}\mathrm{d}v' = \frac{1}{\varepsilon_0}\int_v \rho(\vec{r}',t)\delta\cdot(\vec{R})\mathrm{d}v' = \rho(\vec{r},t)/\varepsilon_0 \tag{27}$$

即得到麦克斯韦方程组之三。

由式(10)取旋度,得库仑电场遵从

$$\nabla \times \vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int_v \rho(\vec{r}',t)\nabla \times \frac{\vec{R}}{R^3}\mathrm{d}v' = 0 \tag{28}$$

将式(28)与式(8)合起来,得到总电场遵从

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} \tag{29}$$

这就是麦克斯韦方程之四。

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Another Method of Deducing Maxwell Equations

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Abstract: It is well - known that the theoretical basis of macroscopic electrodynamics is Maxwell equations, include which Coulomb law, Biot - Savant law, Faraday law of electromagnetic induction and the hypothesis of induced electric field and displacement current. When the distribution of electric charge and electric current varies with time, if given Coulomb law, Biot - Savant law, Faraday law of electromagnetic, and the hypothesis of induced electric field, through the vector analysis, Maxwell Equations are given without the hypothesis of displacement current.

Key words: maxwell equations; magnetic field ; electric field; vector analysis

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Present Status and Development of Synthesis Technologies for TiC、TiN、Ti(C, N) Powders

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Abstract: The preparation technologies of TiC、TiN、Ti(C, N) powders used home and abroad in recent years are reviewed, and the technical processes and characteristics are analysed. The existing problems and development trends of these technologies are also discussed. By seeking cheaper raw materials and achieving new high temperature technologies by plasma, microwave, laser, and electric arc and combining SHS and MA the low - cast and large - scale preparation of high quality TiC, TiN, Ti(C, N) powders are becoming one of the hot research topics. With the development of research and the improvement of preparation techniques, more convenient, more economic and more efficient preparation techniques will provide TiC, TiN, Ti(C,N) powder a broader prospective application.

Key words: titanium carbide; titanium nitride; titanium carbonitride; powders; synthesis method

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