文章编号:1000-582X(2007)02-0130-06

高阶非线性时滞偏微分方程组的振动性定理*

罗李平[1],欧阳自根[2]

(1.衡阳师范学院 数学系,湖南,衡阳 421008;2.南华大学 数学系,湖南,衡阳 421001)

摘　要:偏泛函微分方程来源于物理学、生物学、工程学等学科领域中众多的数学模型,具有强烈的实际背景.振动性理论作为偏泛函微分方程定性理论的重要分支之一,对其进行研究具有极大的理论意义与实用价值.笔者研究一类高阶非线性时滞偏微分方程组的振动性,利用 Green 定理和 Riccati 变换,获得了该类方程组在两类不同边值条件下所有解振动的若干充分性判据,并通过一些实例加以阐述.所得结果为解决上述学科领域中的实际问题提供了数学理论基础.

关键词:高阶偏微分方程组;非线性;时滞;振动性;Riccati 变换

中图分类号:O175.4　　**文献标识码**:A

1　引言

由于在物理学、生物学、医学、化学、工程学和经济学等自然的和人为的过程中都存在着滞后现象,因而用来描述这些过程的数学模型中也就包含着时滞项,即模型是带有泛函变元的偏微分方程,被统称为偏泛函微分方程.因此偏泛函微分方程(组)的研究能更精确地揭示事物本质,同时能丰富微分方程(组)理论的研究.在过去的十几年中,关于偏泛函微分方程理论的研究引起了人们的极大关注,一些基本的结果已收录于专著[1].近几年来,很多学者对偏泛函微分方程组的振动性进行了研究和探讨[2-10],但关于高阶偏泛函微分方程组的振动性的研究还相对较少[11-13].笔者将着手研究如下的一类高阶非线性时滞偏微分方程组

$$\frac{\partial}{\partial t}\left[r(t)\frac{\partial^{n-1}u_i(x,t)}{\partial t^{n-1}}\right]+q_i(x,t)u_i(x,t)+\sum_{j=1}^{m}\sum_{h=1}^{l}q_{ijh}(x,t)f_{ij}(u_j(x,\sigma_h(t)))=a_i(t\Delta u_i(x,t)+\sum_{j=1}^{m}\sum_{k=1}^{s}a_{ijk}(t)\Delta u_j(x,\rho_k(t)))\quad(E)$$

解的振动性,$n\geqslant 2$ 其中是偶数,$(x,t)\in\Omega\times R_+\equiv G$,$R_+=[0,\infty)$,$\Omega\subset R^M$是具有逐片光滑边界 $\partial\Omega$ 的有界区域,且 $\Delta u_i(x,t)=\sum_{r=1}^{M}\frac{\partial^2u_i(x,t)}{\partial x_r^2}$,$i\in I_m$.

考虑如下两类边值条件:

$$\frac{\partial u_i(x,t)}{\partial N}+g_i(x,t)u_i(x,t)=0,(x,t)\in\partial\Omega\times R_+,i\in I_m;\quad(B_1)$$

$$u_i(x,t)=0\quad(x,t)\in\partial\Omega\times R_+,i\in I_m,\quad(B_2)$$

其中 N 是 $\partial\Omega$ 的单位外法向量,$g_i(x,t)\in C(\partial\Omega\times R_+;R_+)$,$i\in I_m$

在本文的讨论中,总假定下列条件成立:

(H_1) $r(t)\in C^1(R_+;(0,\infty))$,$\lim_{t\to\infty}\int_0^t\frac{1}{r(s)}ds=\infty$

(H_2) $q_i(x,t)\in C(\bar{G};R_+)$,$q_i=\min_{x\in\Omega}\{q_i(x,t)\}$,$i\in I_m$,$q(t)=\min_{i\in I_m}\{q_i(t)\}$;

(H_3) $q_{ijh}(x,t)\in C(\bar{G};R)$,$q_{iih}(x,t)>0$,$q_{ijh}(t)=\min_{x\in\Omega}\{q_{ijh}(x,t)\}$,$\bar{q}_{ijh}(t)=\sup_{x\in\Omega}\{|q_{ijh}(x,t)|\}$,$Q_h(t)=\min_{i\in I_m}\{q_{iih}(t)-\sum_{i=1,j\neq i}^{m}\bar{q}_{jih}(t)\}\geqslant 0$,$i,j\in I_m$,$h\in I_l$;

(H_4) $a_i(t)\in C(R_+;R_+)$,$a_{ijk}(t)\in C(R_+;R)$,$a_{iik}(t)>0$,$A_k(t)=\min_{i\in I_m}\{a_{iik}(t)-\sum_{j=1,j\neq i}^{m}|a_{jik}(t)|\}\geqslant 0$,$i,j\in I_m$,$k\in I_s$;$(H_5)$ $\sigma_h(t)$,$\rho_k(t)\in C(R_+,R_+)$且,$\lim_{t\to\infty}\sigma_h(t)=\lim_{t\to\infty}\rho_k(t)=\infty$,$h\in I_l$,$k\in I_s$;

---

* 收稿日期:2006-10-31
基金项目:国家自然科学基金资助项目(10471086)
作者简介:罗李平(1964-),男,副教授,主要从事偏泛函微分方程振动理论的研究,电话(Tel.):0734-8486659,E-mail:layllp@126.com.

(H_6) $f_{ij}(u)C(R,R)$，$uf_{ij}(u)>0(u\neq0)$，$f_{ij}(u)$ 在 $(0,\infty)$ 上是正的非减的凸函数，$f_{ii}(u)\geqslant f_{ji}(u)(u>0)$，$f_{ii}(u)\leqslant f_{ji}(u)(u<0)$，$i,j\in I_m$，$\frac{f_{ii}(u)}{u}\geqslant M$，$u\neq0$ 且 m 是某正数，$i\in I_m$.

定义　称数值函数 $v:G\to R$ 振动，若对任意正数 μ，存在点 $(x_0,t_0)\in\Omega\times[\mu,\infty)$，使得 $v(x_0,t_0)=0$. 否则，称为非振动的；称向量函数 $G\to R^m$ 振动，若它至少有一个分量作为数值函数是振动的；称向量函数 $uG\to R^m$ 非振动，若它的每一个分量作为数值函数都是非振动的.

2　主要结果

首先，考虑系统(E)，(B_1) 的振动性.

定理 1 若存在 $h_0\in I_l$，使得 $\sigma'_{h_0}(t)\geqslant0$，且

$$\int_{t_0}^{\infty}Q_{h_0}(t)\mathrm{d}t=\infty,\quad t_0>0,\tag{1}$$

则系统(E)，(B_1) 的所有解在 G 内振动.

证明　假设系统(E)，(B_1) 有一个非振动解 $u(x,t)=(u_1(x,t),u_2(x,t),\cdots,u_m(x,t))^{\mathrm{T}}$，不妨设当 $t\geqslant t_0>0$ 时，$|u_i(x,t)|>0$，$i\in I_m$. 令 $\delta_i=\mathrm{sgn}u_i(x,t)$，$Z_i(x,t)=\delta_lu_i(x,t)$，则 $Z_i(x,t)>0$，$(x,t)\in\Omega\times[t_0,\infty)$，$i\in I_m$. 由条件 (H_5) 知，存在 $t_1\geqslant t_0$，使得 $Z_i(x,t)>0$，$Z_j(x,\rho_k(t))>0$，$Z_j(x,\sigma_h(t))>0$，$(x,t)\in\Omega\times[t_1,\infty)$，$i,j\in I_m$，$k\in I_s$，$h\in I_l$.

对方程(E)两边关于 x 在 Ω 上积分有

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[r(t)\frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}}\int_\Omega Z_i(x,t)\mathrm{d}x\right]=$$
$$a_i(t)\int_\Omega\Delta Z_i(x,t)\mathrm{d}x+\sum_{j=1}^{m}\sum_{k=1}^{s}a_{ijk}(t)\frac{\delta_j}{\delta_i}\int_\Omega\Delta Z_i(x,\rho_k(t))\mathrm{d}x-$$
$$\int_\Omega q_i(x,t)Z_i(x,t)\mathrm{d}x-\sum_{j=1}^{m}\sum_{h=1}^{l}\frac{1}{\delta_i}$$
$$\int_\Omega q_{ijh}(x,t)f_{ij}(\delta_jZ_j(x,\sigma_h(t)))\mathrm{d}x,t\geqslant t_1,i\in I_m.\tag{2}$$

由 Green 公式及边值条件 (B_1) 有

$$\int_\Omega\Delta Z_i(x,t)\mathrm{d}x=\int_{\partial\Omega}\frac{\partial Z_i(x,t)}{\partial N}\mathrm{d}S=$$
$$-\int_{\partial\Omega}g_j(x,t)Z_i(x,t)\mathrm{d}S\leqslant0,t\geqslant t_1,i\in I_m.\tag{3}$$

$$\int_\Omega\Delta Z_j(x,\rho_k(t))\mathrm{d}x=-\int_{\partial\Omega}g_j(x,\rho_k(t))\ ,$$
$$Z_j(x,\rho_k(t))\mathrm{d}S,\quad t\geqslant t_1,j\in I_m,k\in I_s,\tag{4}$$

其中 $\mathrm{d}S$ 是 $\partial\Omega$ 上的面积元素.

又据条件 (H_2)，(H_3)，(H_6) 及 Jensen 不等式有

$$\int_\Omega q_i(x,t)Z_i(x,t)\mathrm{d}x\geqslant q_i\int_\Omega Z_i(x,t)\mathrm{d}x,t\geqslant t_1,i\in I_m,\tag{5}$$

$$\int_\Omega q_{ijh}(x,t)f_{ij}(\delta_jZ_j(x,\sigma_h(t)))\mathrm{d}x\geqslant q_{ijh}(t)\left(\int_\Omega\mathrm{d}x\right)f_{ij}$$
$$\left(\delta_j\left(\int_\Omega\mathrm{d}x\right)^{-1}\int_\Omega Z_i(x,\sigma_h(t))\mathrm{d}x\right),t\geqslant t_1,i,j\in I_m,h\in I,\tag{6}$$

令 $V_i(t)=\left(\int_\Omega\mathrm{d}x\right)^{-1}\int_\Omega Z_i(x,t)\mathrm{d}x$，$Y_j(t)=\left(\int_\Omega\mathrm{d}x\right)^{-1}\int_{\partial\Omega}g_j(x,t)Z_j(x,t)\mathrm{d}S$，$t\geqslant t_1$，$i,j\in I_m$，则 $V_i(t)>0$，$Y_j(t)\geqslant0$，$t\geqslant t_1$，$i,j\in I_m$. 于是由(2)-(6)可得

$$[b(t)V_i^{(n-1)}(t)]'\leqslant-\sum_{k=1}^{s}\Big\{a_{iik}(t)Y_i(\rho_k(t))-$$
$$\sum_{j=1,j\neq i}^{m}|a_{ijk}(t)|Y_j(\rho_k(t))\Big\}-q_i(t)V_i(t)-$$
$$\sum_{h=1}^{l}\Big\{q_{iih}(t)\frac{f_{ii}(\delta_iV_i(\sigma_h(t)))}{\delta_i}-$$
$$\sum_{j=1,j\neq i}^{m}\bar{q}_{ijh}(t)\frac{f_{ij}(\delta_jV_j(\sigma_h(t)))}{\delta_j}\Big\},t\geqslant t_1,i\in I_m.$$

令 $V(t)=\sum_{i=1}^{m}V_i(t)$，$Y(t)=\sum_{j=1}^{m}Y_j(t)$，$t_1\geqslant t_1$，则 $V(t)>0$，$Y(t)\geqslant0$，$t\geqslant t_1$. 上式按 $i=1,2,\cdots,m$ 垂直相加，并结合条件 (H_3)，(H_4) 得

$$[r(t)V_i^{(n-1)}(t)]'\leqslant-\sum_{k=1}^{s}\Big\{\sum_{i=1}^{m}[a_{iik}(t)Y_i(\rho_k(t))-$$
$$\sum_{j=1,j\neq i}^{m}|a_{ijk}(t)|Y_j(\rho_k(t))]\Big\}-q(t)V(t)-$$
$$\sum_{h=1}^{l}\Big\{\sum_{i=1}^{m}[q_{iih}(t)\frac{f_{ii}(\delta_iV_i(\sigma_h(t)))}{\delta_i}-$$
$$\sum_{j=1,j\neq i}^{m}\bar{q}_{ijh}(t)\frac{f_{ij}(\delta_jV_j(\sigma_j(t)))}{\delta_i}]\Big\}=-$$
$$\sum_{k=1}^{s}\Big\{\sum_{i=1}^{m}[a_{iik}(t)-\sum_{j=1,j\neq i}^{m}|a_{jik}(t)|]Y_i(\rho_k(t))\Big\}-$$
$$q(t)V(t)-\sum_{h=1}^{l}\Big\{\sum_{i=1}^{m}\Big[q_{iih}(t)-\sum_{j=1,j\neq i}^{m}\bar{q}_{jih}(t)\Big]$$
$$\frac{f_{ii}(\delta_iV_i(\sigma_h(t)))}{\delta_i}\Big\}\leqslant-\sum_{k=1}^{s}\min_{i\in I_m}\Big\{a_{iik}(t)-$$
$$\sum_{j=1,j\neq i}^{m}|a_{jik}(t)|\Big\}\sum_{i=1}^{m}Y_i(\rho_k(t))-q(t)V(t)-$$
$$\sum_{h=1}^{l}\min_{i\in I_m}\Big\{q_{iih}(t)-\sum_{j=1,j\neq i}^{m}\bar{q}_{jih}(t)\Big\}\sum_{i=1}^{m}\frac{f_{ii}[\delta_iV_i(\sigma_h(t))]}{\delta_iV_i(\sigma_h(t))}$$
$$V_i(\sigma_h(t))\leqslant-q(t)V(t)-\sum_{k=1}^{s}A_k(t)Y(\rho_k(t))-$$
$$\sum_{h=1}^{l}\bar{Q}_h(t)V(\sigma_h(t))$$

其中 $\bar{Q}_h(t)=MQ_h$，$t\geqslant t_1$.

因此有

$$[r(t)V^{(n-1)}(t)]'+q(t)V(t)+\sum_{k=1}^{s}A_k(t)Y(\rho_k(t))+$$

$$\sum_{h=1}^{l}\overline{Q}_h(t)V(\sigma_h(t)) \leqslant 0, \quad t \geqslant t_1. \qquad (7)$$

而 $Y(\rho_k(t))\geqslant 0, t\geqslant t_1, k\in I_s$，于是由(7)有

$$[r(t)V^{(n-1)}(t)]' + q(t)V(t) + \sum_{h=1}^{l}\overline{Q}_h(t)V(\sigma_h(t)) \leqslant 0, \quad t \geqslant t_1. \qquad (8)$$

由(8)可知 $[r(t)V^{(n-1)}(t)]'\leqslant 0, t\geqslant t_1$，所以 $r(t)V^{(n-1)}(t)$ 单调减少，从而可推得

$$r(t)V^{(n-1)}(t) \geqslant 0, t \geqslant t_1. \qquad (9)$$

事实上，若(9)不成立，则必存在 $T\geqslant t_1$，满足 $r(T)V^{(n-1)}(T)<0$，所以当 $t\geqslant T$ 时，由 $r(t)V^{(n-1)}(t)$ 单调减少知，$r(t)V^{(n-1)}(t)\leqslant r(T)V^{(n-1)}(T)$，从而 $V^{(n-1)}(t)\leqslant\frac{1}{r(t)}r(T)V^{(n-1)}(T), t\geqslant T$. 因此

$$V^{(n-2)}(t) - V^{(n-2)}(T) \leqslant r(T)^{(n-1)}(T)\int_T^t \frac{1}{r(s)}\mathrm{d}s, t \geqslant T.$$

在上式中，令 $t\to\infty$，得到 $\lim\limits_{t\to\infty}V^{(n-2)}(t) = -\infty$，同理类推，可得

$$\lim_{t\to\infty}V^{(k)}(t) = -\infty, \qquad k = 0,1,2,\cdots,n-2,$$

但这与“$V^{(t)}>0$”矛盾，故(9)成立.

特别地，由 $r(t)>0$ 及(9)得 $V^{(n-1)}(t)\geqslant 0, t\geqslant t_1$ 进而可推得

$$V'(t) \geqslant 0, t \geqslant t_1. \qquad (10)$$

若不成立，即 $V'(t)<0$，则由 n 是偶数及著名的 Kiguradze 引理[14]可得，对更高阶的奇数阶导数有

$$V^{(k)}(t) < 0, \quad k = 3,5,\cdots,n-1.$$

但这与上述结果“$V^{(n-1)}(t)\geqslant 0$”矛盾，故(10)成立.

由(8)有

$$[r(t)V^{(n-1)}(t)]' + \overline{Q}_{h_0}(t)V(\sigma_{h_0}(t)) \leqslant 0, t \geqslant t_1. \qquad (11)$$

令 $W(t)=\frac{r(t)V^{(n-1)}(t)}{V(\sigma_{h_0}(t))}, t\geqslant t_1$，则 $W(t)\geqslant 0, t\geqslant t_1$，且结合(11)有

$$W'(t) = \frac{[r(t)V^{(n-1)}(t)]'}{V(\sigma_{h_0}(t))} - \frac{r(t)V^{(n-1)}(t)V'(\sigma_{h_0}(t))\sigma'_{h_0}(t)}{V^2(\sigma_{h_0}(t))} \leqslant -\overline{Q}_{h_0}(t).$$

对上述不等式从 t_1 到 $t(t>t_1)$ 积分得

$$0 \leqslant W(t) \leqslant W(t_1) - \int_{t_1}^{t}\overline{Q}_{h_0}(s)\mathrm{d}s.$$

令 $t\to\infty$，由(1)可得 $\lim\limits_{t\to\infty}W(t) = -\infty$，但这与“$W(t)\geqslant 0$”，故定理 1 得证.

推论 1 若将定理 1 中的条件(1)换为

$$\int_{t_0}^{\infty} q(t)\mathrm{d}t = \infty \qquad t_0 > 0,$$

则系统(E)，(B_1)的所有解在 G 内振动.

证明 如同在定理 1 的证明中，由微分不等式(8)有

$$[r(t)V^{(n-1)}(t)]' + q(t)V(t) \leqslant 0, \quad t \geqslant t_1.$$

余下的证明与定理 1 类似，故省略. 证毕.

推论 2 若微分不等式(8)无最终正解，则系统(E)，(B_1)的所有解在 G 内振动.

定理 2 若存在 $h_0\in I_l$，使得 $\sigma'_{h_0}(t)\geqslant 0$，且存在单调递增函数 $\varphi(t)\in C^1([t_0,\infty],(0,\infty))$，使得对任意正常数 β，有

$$\int_{t_0}^{\infty}[\varphi(t)\overline{Q}_{h_0}(t) - \beta\varphi'(t)]\mathrm{d}t = \infty, t_0 > 0, \qquad (12)$$

则系统(E)，(B_1)的所有解在 G 内振动.

证明 如同在定理 1 的证明中，可得

$$[r(t)V^{(n-1)}(t)]' \leqslant 0, V^{(n-1)}(t) \geqslant 0,$$
$$V(t) > 0, V'(t) \geqslant 0, t \geqslant t_1,$$

且有(11).

令，$H(t) = \frac{\varphi(t)r(t)V^{(n-1)}(t)}{V(\sigma_{h_0}(t))}, t \geqslant t_1$ 则 $H(t)\geqslant 0, t\geqslant t_1$ 且结合(11)有

$$W'(t) = \frac{\varphi'(t)r(t)V^{(n-1)}(t)}{V(\sigma_{h_0}(t))} + \frac{\varphi(t)[r(t)V^{(n-1)}(t)]'}{V(\sigma_{h_0}(t))} - \frac{\varphi(t)r(t)V^{(n-1)}(t)V'(\sigma_{h_0}(t))\sigma'_{h_0}(t)}{V^2(\sigma_{h_0}(t))} \leqslant$$
$$\frac{\varphi'(t)r(t)V^{(n-1)}(t)}{V(\sigma_{h_0}(t))} + \frac{\varphi(t)[r(t)V^{(n-1)}(t)]'}{V(\sigma_{h_0}(t))} \leqslant$$
$$\frac{\varphi'(t)r(T)V^{(n-1)}(T)}{V(\sigma_{h_0}(T))} - \varphi(t)\overline{Q}_{h_0}(t) =$$
$$\beta_0\varphi'(t) - \varphi(t)\overline{Q}_{h_0}(t),$$

其中 $\beta_0=\frac{r(T)V^{(n-1)}(T)}{V(\sigma_{h_0}(T))}>0, T\geqslant t_1$ 且 $V'(\sigma_{h_0}(t))>0$，$t\geqslant T$.

对上述不等式从 T 到 $t(t>T)$ 积分得

$$0 \leqslant H(t) \leqslant H(T) - \int_{t_1}^{t}[\varphi(s)\overline{Q}_{h_0}(s) - \beta_0\varphi'(s)]\mathrm{d}s.$$

令 $t\to\infty$，由(12)可得 $\lim\limits_{t\to\infty}H(t) = -\infty$，但这与“$H(t\geqslant 0)$”矛盾，故定理 2 得证.

下面考虑系统(E)，(B_2)的振动性. 为此，在 Ω 上考虑 Dirichlet 问题：

$$\begin{cases}\Delta\omega(x) + \lambda\omega(x) = 0, & x \in \Omega \\ \omega(x) = 0, & x \in \partial\Omega\end{cases}, \qquad (13)$$

其中 λ 是常数.

令 λ_0 是问题(13)的最小特征值，则据文献[15]知，$\lambda_0>0$，且 $\forall x\in\Omega$，其相应的特征函数 $\varphi(x)>0$.

定理 3 若定理 1 中的所有条件成立，则系统(E)，(B_2)的所有解在 G 内振动.

证明　假设系统(E)，(B_2)有一个非振动解 $u(x,t)=(u_1(x,t),u_2(x,t),\cdots,u_m(x,t))^{T}$，不妨设当 $t\geqslant t_0>0$ 时 $|u_i(x,t)|>0, i\in I_m$. 令 $\delta_i=\operatorname{sgn}u_i(x,t)$，$Z_i(x,t)=\delta_i u_i(x,t)$ 则 $Z_i(x,t)>0,(x,t)\in\Omega\times[t_0,\infty), i\in I_m$. 由条件($H_5$)知，存在 $t_1\geqslant t_0$，使得 $Z_i(x,t)>0, Z_j(x,\rho_k(t))>0, Z_j(x,\sigma_h(t))>0,(x,t)\in\Omega\times[t_1,\infty), i,j\in I_m, k\in I_s, h\in I_l$.

在方程(1)两边同乘以 $\varphi(x)$ 并在 Ω 上关于积分，可得

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[r(t)\frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}}\int_\Omega Z_i(x,t)\varphi(x)\mathrm{d}x\right]=$$
$$a_i(t)\int_\Omega \Delta Z_i(x,t)\varphi(x)\mathrm{d}x+\sum_{j=1}^{m}\sum_{k=1}^{s}a_{ijk}(t)\frac{\delta_j}{\delta_i}$$
$$\int_\Omega \Delta Z_j(x,\rho_k(t))\varphi(x)\mathrm{d}x=\int_\Omega q_i(x,t)Z_i(x,t)\varphi(x)\mathrm{d}x-$$
$$\sum_{j=1}^{m}\sum_{h=1}^{l}\frac{1}{\delta_i}\int_\Omega q_{ijh}(x,t)f_{ij}(\delta_j Z_j(x,\sigma_h(t)))\varphi(x)\mathrm{d}x,$$
$$t\geqslant t_1, i\in I_m. \tag{14}$$

由 Green 公式及边值条件(B_2)有

$$\int_\Omega \Delta Z_i(x,t)\varphi(x)\mathrm{d}x=\int_\Omega Z_i(x,t)\Delta\varphi(x)\mathrm{d}x=$$
$$-\lambda_0\int_\Omega Z_i(x,t)\varphi(t)\mathrm{d}x\leqslant 0, t\geqslant t_1, i\in I_m, \tag{15}$$
$$\int_\Omega \Delta Z_j(x,\rho_k(t))\varphi(x)\mathrm{d}x=$$
$$-\lambda_0\int_\Omega Z_j(x,\rho_k(t))\varphi(t)\mathrm{d}x\leqslant 0,$$
$$t\geqslant t_1, j\in I_m, k\in I_s. \tag{16}$$

又据条件(H_2)，(H_3)，(H_6)及 Jensen 不等式有

$$\int_\Omega q_i(x,t)Z_i(x,t)\varphi(x)\mathrm{d}x\geqslant$$
$$q_i(t)\int_\Omega Z_i(x,t)\varphi(x)\mathrm{d}x, t\geqslant t_1, i\in I_m. \tag{17}$$
$$\int_\Omega q_{ijh}(x,t)f_{ij}(\delta_j Z_j(x,\sigma_h(t)))\varphi(x)\mathrm{d}x\geqslant$$
$$q_{ijh}(t)\left(\int_\Omega\varphi(x)\mathrm{d}x\right)f_{ij}\Big(\delta_j\Big(\int_\Omega\varphi(x)\mathrm{d}x\Big)^{-1}$$
$$\int_\Omega Z_i(x,\sigma_h(t))\varphi(x)\mathrm{d}x\Big), t\geqslant t_1, i,j\in I_m, h\in I_l. \tag{18}$$

令 $U_i(t)=\left(\int_\Omega\varphi(x)\mathrm{d}x\right)^{-1}\int_\Omega Z_i(x,t)\varphi(x)\mathrm{d}x$，$t\geqslant t_1, i\in I_m$，则 $U_i(t)>0, t\geqslant t_1, i\in I_m$. 于是由(14)—(18)可得

$$[b(t)U_i^{(n-1)}(t)]'\leqslant-\lambda_0\sum_{k=1}^{s}\Big\{a_{iik}(t)U_i(\rho_k(t))-$$
$$\sum_{j=1,j\neq i}^{m}|a_{ijk}(t)|U_j(\rho_k(t))\Big\}-q_i(t)U_i(t)-$$
$$\sum_{h=1}^{l}\Big\{q_{iih}(t)\frac{f_{ii}(\delta_i U_i(\sigma_h(t)))}{\delta_i}-$$
$$\sum_{j=1,j\neq i}^{m}\bar q_{ijh}(t)\frac{f_{ij}(\delta_j U_j(\sigma_h(t)))}{\delta_j}\Big\}, t\geqslant t_1, i\in I_m.$$

令 $U(t)=\sum_{i=1}^{m}U_i(t), t\geqslant t_1$，则 $U(t)>0, t\geqslant t_1$. 上式按 $i=1,2,\cdots,m$ 垂直相加，并结合条件(H_3)，(H_4)得

$$[r(t)U^{(n-1)}(t)]'\leqslant-\lambda_0\sum_{k=1}^{s}\Big\{\sum_{i=1}^{m}\Big[a_{iik}(t)U_i(\rho_k(t))-$$
$$\sum_{j=1,j\neq i}^{m}|a_{ijk}(t)|U_j(\rho_k(t))\Big]\Big\}-\sum_{h=1}^{l}\Big\{\sum_{i=1}^{m}\Big[q_{iih}(t)$$
$$\frac{f_{ii}(\delta_i U_i(\sigma_h(t)))}{\delta_i}-\sum_{j=1,j\neq i}^{m}\bar q_{ijh}(t)\frac{f_{ij}(\delta_j U_j(\sigma_h(t)))}{\delta_j}\Big]\Big\}=$$
$$-\lambda_0\sum_{k=1}^{s}\Big\{\sum_{i=1}^{m}\Big[a_{iik}(t)-\sum_{j=1,j\neq i}^{m}|a_{jik}(t)|\Big]U_i(\rho_k(t))\Big\}-$$
$$q(t)U(t)-\sum_{h=1}^{l}\Big\{\sum_{i=1}^{m}\Big[q_{iih}(t)-\sum_{j=1,j\neq i}^{m}\bar q_{ijh}(t)\Big]$$
$$\frac{f_{ii}(\delta_i U_i(\sigma_h(t)))}{\delta_i}\Big\}\leqslant-\lambda_0\sum_{k=1}^{s}\min_{i\in I_m}\Big\{q_{iih}(t)-$$
$$\sum_{j=1,j\neq i}^{m}\bar q_{jih}(t)\Big\}\sum_{i=1}^{m}\frac{f_{ii}[\delta_i U_i(\sigma_h(t))]}{\delta_i U_i(\sigma_h(t))}U_i(\sigma_h(t))\leqslant$$
$$-q(t)V(t)-\lambda_0\sum_{k=1}^{s}A_k(t)U(\rho_k(t))-\sum_{h=1}^{l}\bar Q_h(t)U(\sigma_h(t)).$$

其中 $\bar Q_h(t)=MQ_h(t), t\geqslant t_1$.

因此有

$$[r(t)U(n-1)(t)]'+q(t)U(t)+$$
$$\lambda_0\sum_{k=1}^{s}A_k(t)U(\rho_k(t))+\sum_{h=1}^{l}\bar Q_h(t)U(\sigma_h(t))\leqslant 0,$$
$$t\geqslant t_1. \tag{19}$$

余下的证明类似于定理 1 后半部分证明，故省略. 证毕

推论 3　若微分不等式(19)无最终正解，则系统(E)，(B_2)的所有解在 G 内振动.

由微分不等式(19)有

$$[r(t)U^{(n-1)}(t)]'+\lambda_0\sum_{k=1}^{s}A_k(t)U(\rho_k(t))\leqslant 0, t\geqslant t_1.$$

类似于定理 1 的证明，可得如下的推论.

推论 4　若存在 $k\in I_s$，使得 $\rho'_{k_0}(t)\geqslant 0$ 且

$$\int_{t_0}^{\infty}\lambda_0 A_{k_0}(t)\mathrm{d}t=\infty,\quad t_0>0,$$

则系统(E)，(B_2)的所有解在 G 内振动.

由微分不等式(19)有

$$[r(t)U^{(n-1)}(t)]'+q(t)U(t)\leqslant 0,\quad t\geqslant t_1.$$

类似于定理 1 的证明，可得如下的推论.

推论 5　若推论 1 中的所有条件成立，则系统(E)，(B_2)的所有解在 G 内振动.

3 应用举例

例 1 考虑偏微分方程组

$$\begin{cases}\frac{\partial}{\partial t}\left[t\frac{\partial^5 u_1(x,t)}{\partial t^5}\right]+(1+t)u_1(x,t)+\\3u_1(x,t-\pi)+u_2(x,t-\pi)+u_1(x,t-\frac{\pi}{2})+\\u_2(x,t-\frac{\pi}{2})=\Delta u_1(x,t)+\Delta u_1(x,t-\frac{3\pi}{2}),\\\frac{\partial}{\partial t}\left[t\frac{\partial^5 u_2(x,t)}{\partial t^5}\right]+(2+t)u_2(x,t)+\\2u_1(x,t-\pi)+\frac{3}{2}u_2(x,t-\pi)+u_1(x,t-\frac{\pi}{2})+\\3u_2(x,t-\frac{\pi}{2})=\Delta u_2(x,t)+\frac{1}{2}\Delta u_2(x,t-\frac{3\pi}{2}),\end{cases}$$

$$(x,t)\in(0,\pi)\times[0,\infty)=G, \quad (20)$$

边值条件为

$$\frac{\partial}{\partial x}u_i(0,t)=\frac{\partial}{\partial x}u_i(\pi,t)=0,t\geqslant 0,i=1,2. \quad (21)$$

不难验证它满足定理 1 的全部条件，所以系统(20)，(21)的所有解在 G 上振动. 事实上，$u_1(x,t)=\cos x\sin t$，$u_2(x,t)=\cos x\cos t$，就是这样的一个解.

例 2 考虑偏微分方程组

$$\begin{cases}\frac{\partial}{\partial t}\left[t\frac{\partial^5 u_1(x,t)}{\partial t^5}\right]+(1+t)u_1(x,t)+\\2u_1(x,t-\pi)+u_2(x,t-\pi)+3u_1(x,t-\frac{\pi}{2})+\\u_2(x,t-\frac{\pi}{2})=2\Delta u_1(x,t)+\Delta u_1(x,t-\frac{3\pi}{2}),\\\frac{\partial}{\partial t}\left[t\frac{\partial^5 u_2(x,t)}{\partial t^5}\right]+(\frac{1}{2}+t)u_2(x,t)+\\u_1(x,t-\pi)+3u_2(x,t-\pi)+u_1(x,t-\frac{\pi}{2})+\\\frac{3}{2}u_2(x,t-\frac{\pi}{2})=\frac{3}{2}\Delta u_2(x,t)+\frac{3}{2}\Delta u_2(x,t-\frac{3\pi}{2}),\end{cases}$$

$$(x,t)\in(0,\pi)\times[0,\infty)=G, \quad (22)$$

边值条件为

$$u_i(0,t)=u_i(\pi,t)=0,t\geqslant 0,i=1,2. \quad (23)$$

不难验证它满足定理 1 的全部条件，所以系统(22)，(23)的所有解在 G 上振动. 事实上 $u_1(x,t)=\sin x\cos t$，$u_2(x,t)=\sin x\sin t$ 就是这样的一个解.

参考文献：

[1] WU J H. Theory and applications of partial functional differential equations[M]. New York: Springs-Verlag, 1996.

[2] 罗李平. 一类拟线性抛物型偏微分方程系统解的振动性[J]. 哈尔滨师范大学自然科学学报, 2005, 21(5): 31-33, 37.

[3] 李伟年, 孟凡伟. 偏泛函微分方程系统解的强迫振动性[J]. 系统科学与数学, 2004, 24(3): 324-331.

[4] LI W N, CUI B T. Oscillation for systems of neutral delay hyperbolic differential equations[J]. Indian J Pure Appl Math, 2001, 31: 933-948.

[5] LI W N, MENG F W. Oscillation for systems of neutral partial differential equations with continuous distributed deviating arguments[J]. Demonstratio Math, 2001, 34: 619-633.

[6] 邓立虎, 葛渭高, 俞元洪. 拟线性抛物泛微分方程组有关边值问题的振动性[J]. 应用数学学报, 2001, 24(2): 295-301.

[7] 关新平, 杨军. 非线性中立型双曲偏泛函微分方程系统的振动性[J]. 系统科学与数学, 1998, 18(2): 239-246.

[8] 李伟年. 时滞抛物方程组解的振动性定理[J]. 数学的实践与认识, 2001, 31(2): 143-147.

[9] 李永昆. 具有偏差变元的双曲型微分方程组解的振动性[J]. 数学学报, 1997, 40(1): 100-105.

[10] YANG J, GUAN X P. Oscillation for sysrems of nonlinear neutral type parabolic partial functional differential equations[J]. Appl Math-JCU., 1997, 12B(2): 165-178.

[11] 林文贤. 关于高阶中立型偏微分方程系统解的振动性[J]. 应用数学学报, 2005, 28(3): 571-573.

[12] LIN W X. Oscillation for solutions of systems of n-th order partial functional differential equations[J]. Ann of Diff Eqs, 2004, 20(3): 266-272.

[13] 林文贤. 高阶拟线性中立型偏泛函微分方程组解的振动性[J]. 高等学校计算数学学报, 2003, 25(1): 50-59.

[14] ERBE L H, QINGKAI KONGg, ZHANG B G. Oscillation theory for functional differential equation[M]. New York: Marcel Dekker, 1995: 289.

[15] VLADIMIROV V S. Equations of Mathematical Physics[M]. Moscow: Nauka, 1981.

Oscillation Theorems of Systems of High Order Nonlinear Delay Partial Differential Equations

LUO Li-ping[1], OUYANG Zi-gen[2]

(1. Department of Mathematics, Hengyang Normal University, Hengyang, Hunan 421008, China;
2. Department of Mathematics, Nanhua University, Hengyang, Hunan 421001, China)

Abstract: Partial functional differential equations come from many mathematical models in physics, biology, engineering and other fields, which have strongly practical background. The oscillation theory is the one of the important branches of qualitative theory of partial functional differential equations. Therefore, it is of great theoretical and practical value to research the oscillation of partial functional differential equations. The anthors study the oscillation of the systems of a class of high order nonlinear delay partial functional differential equations. By using Green's theorem and Riccati transformation, they obtain some sufficient criteria for oscillation of all solutions of the systems under two kinds of different boundary value conditions, which are illustrated by some examples. These results offer the foundation of mathematical theory for solving the practical problems of the above fields.

Key words: system of high order partial differential equation; nonlinear; delay; oscillation; Riccati transformation

(编辑　张小强)

(上接第 129 页)

Reaction Time of Road Lighting in the Mesopic Vision Condition

ZHANG Qing-wen, CHEN Zhong-lin, LIU Ying-ying

(College Architecture and Urban Planning, Chongqing University, Chongqing 400030, China)

Abstract: Brightness level of the road lighting should be of the mesopic vision category. But the present road lighting standard in the various countries is in the bright vision condition, this is different with the mesopic vision effect, which will bring the appraisal error into the illumination quality. Therefore, studies on the mesopic vision condition through the reaction time of the road lighting have been the front research topic. Regarding this, this article introduces the middle vision reaction time test system which independently develops to carry on the photosource light effect the research. This system can adopt different light sources to form the illumination environment under the mesopic vision, simulate road barrier ahead the car at night with varied light, and examine reaction time according to visual characteristic of driver through electronic shutter. Through regression analysis of the massive measured data, the reaction time and the background brightness function expression has been fitted under the different experimental condition, thus obtains the golden halogen photosource vision effect to be higher than the sodium lamp photosource the conclusion. In the article, the system the structure, the principle of work, the experimental technique and the data processing result has carried on the analysis and the discussion. This research results which is founded in the mesopic visual condition the visibility model provide the experimental basis, which has the very good model significance for China road lighting project concrete implementation and the actual improvement.

Key words: mesopic vision; reaction time; background luminance; contrast ratio depended on mark; partial horn depended on mark; coefficient correlation

(编辑　姚　飞)