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Hammerstein系统遗忘因子有限窗口分解辨识

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摘要: 提出一种带遗忘因子和分解辨识策略的有限数据窗口递归最小二乘Hammerstein系统辨识方法。针对Hammerstein系统具有耦合参数的问题,将Hammerstein系统分解为2个子系统:一个子系统包含线性子系统参数,另一个子系统包含非线性子系统参数;提出一种基于遗忘因子的有限窗口递归最小二乘方法对分解模型进行在线递归估计;仿真示例验证了所提算法能够快速跟踪参数,实现对Hammerstein系统的精确辨识。

关键词: Hammerstein系统;递归辨识;最小二乘法;遗忘因子

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Identification of Hammerstein systems using decomposition based finite-data-window recursive least squares method with a forgetting factor

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Abstract: In this paper, a decomposition based recursive finite-data-window least squares identification method with a forgetting factor is proposed for Hammerstein systems. The proposed method aims to identify the parameters of Hammerstein systems by decomposing them into two subsystems, one involving linear subsystem parameters, and the other containing the nonlinear subsystem parameters. To achieve this, a two-step finite-data-window recursive least squares method with a forgetting factor is developed. To verify the effectiveness and merits of the proposed algorithm, a simulation example is provided, demonstrating that the proposed algorithm can quickly track parameters and accurately and effectively identify Hammerstein systems.

Keywords: Hammerstein systems; recursive identification; least-squares method; forgetting factor

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Hammerstein模型由一个非线性无记忆部分和一个线性动态系统组成,在许多工程问题中得到广泛应用,因此多年以来,Hammerstein模型辨识一直是一个活跃的研究领域[1-6]。辨识Hammerstein系统有很多方法[7-14],包括过参数化法、随机法、可分离最小二乘法、盲法、频域法、子空间法、迭代法和递阶法等。

笔者重点研究了基于递阶辨识原理的方法[8],将Hammerstein模型分解为2个子系统:一个子系统包含线性子系统参数,另一个子系统包含非线性子系统参数。基于递阶辨识原理的方法的特点是这2个子系统包含的未知参数最少,与过参数化方法相比具有更高计算效率。大量递阶Hammerstein模型辨识方法已被相继提出[15-18]。

基于递阶辨识原理,提出基于遗忘因子[19]和有限数据窗的最小二乘法算法,对Hammerstein模型进行递归辨识。该方法同时保留了遗忘因子最小二乘法和有限数据窗最小二乘法的优点,因此,该方法能够快速跟踪估计参数。提出的基于递阶原理的Hammerstein模型辨识方法的优点在于它能够快速地跟踪Hammerstein模型中的参数,并且比基于过参数化模型的算法具有更高计算效率[20-22]。

1　问题描述

考虑如图1所示的Hammerstein非线性系统,由下式描述:

$$y(t)=-\sum_{i=1}^{n_a}a_i y(t-i)+\sum_{i=1}^{n_b}b_i\bar{u}(t-i)+v(t),\tag{1}$$

$$\bar{u}(t)=f\big(u(t)\big)=\sum_{i=1}^{m}c_i f_i\big(u(t)\big)。\tag{2}$$

式中:$u(t)$和$y(t)$分别是t时刻系统的输入数据和输出数据;$v(t)$是零均值的不相关的随机噪声;$\bar{u}(t)$是系统内部的过程变量,由未知系数c_i和已知基函数$\boldsymbol{f}(\cdot)=\begin{bmatrix}f_1(\cdot) & f_2(\cdot) & \cdots & f_m(\cdot)\end{bmatrix}$的线性组合描述,其中$m$是基函数$\boldsymbol{f}(\cdot)$的条目;$n_a$和$n_b$分别是多项式$A(z)$和$B(z)$的阶次,其中$A(z)=1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_a}z^{-n_a}$,$B(z)=b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+\cdots+b_{n_b}z^{-n_b}$。本研究的任务是通过利用输入数据和输出数据来辨识未知参数a_i,b_i和c_i。

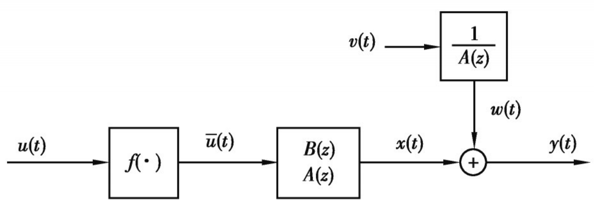

图1　Hammerstein系统

Fig. 1　The Hammerstein system

2　研究方法

用单位推移算子z^{-1}来重写式(1)和式(2)(即$z^{-1}y(t)=y(t-1)$),得到

$$A(z)y(t)=B(z)\bar{u}(t)+v(t),\tag{3}$$

$$\bar{u}(t)=f\big(u(t)\big)=\boldsymbol{f}\big(u(t)\big)\boldsymbol{c}。\tag{4}$$

定义线性子系统的参数向量$\boldsymbol{a}$、$\boldsymbol{b}$和非线性部分的参数向量$\boldsymbol{c}$如下:

$$\boldsymbol{a}=\begin{bmatrix}a_1 & a_2 & \cdots & a_{n_a}\end{bmatrix}^{\mathrm{T}}\in\mathbf{R}^{n_a},$$

$$\boldsymbol{b}=\begin{bmatrix}b_1 & b_2 & \cdots & b_{n_b}\end{bmatrix}^{\mathrm{T}}\in\mathbf{R}^{n_b},$$

$$\boldsymbol{c}=\begin{bmatrix}c_1 & c_2 & \cdots & c_m\end{bmatrix}^{\mathrm{T}}\in\mathbf{R}^{m},$$

$$\boldsymbol{\Theta}=\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\\ \boldsymbol{c}\end{bmatrix}\in\mathbf{R}^{n_a+n_b+m}。$$

关于参数向量 $\boldsymbol{c}$，给出如下假设。

假设 1.$\|\boldsymbol{c}\|=1$，并且向量 $\boldsymbol{c}$ 的第一项是正的。也就是说，函数 $f(\cdot)$ 的第一个系数是正的，即 $c_1>0$。

基于式(3)和式(4)，可得

$$y(t)=-\sum_{i=1}^{n_a}a_i y(t-i)+\sum_{i=1}^{n_b}b_i \boldsymbol{f}\left(u(t-i)\right)\boldsymbol{c}+v(t)=\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{a}+\boldsymbol{b}^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{c}+v(t)。\tag{5}$$

式中：

$$\boldsymbol{\varphi}(t)=\begin{bmatrix}-y(t-1) & y(t-2) & \cdots & -y(t-n_a)\end{bmatrix}^{\mathrm{T}}\in\mathbf{R}^{n_a},$$

$$\boldsymbol{F}(t)=\begin{bmatrix}\boldsymbol{f}\left(u(t-1)\right)\\ \boldsymbol{f}\left(u(t-2)\right)\\ \vdots\\ \boldsymbol{f}\left(u(t-n_b)\right)\end{bmatrix}=\begin{bmatrix}f_1\left(u(t-1)\right) & f_2\left(u(t-1)\right) & \cdots & f_m\left(u(t-1)\right)\\ f_1\left(u(t-2)\right) & f_2\left(u(t-2)\right) & \cdots & f_m\left(u(t-2)\right)\\ \vdots & \vdots & & \vdots\\ f_1\left(u(t-n_b)\right) & f_2\left(u(t-n_b)\right) & \cdots & f_m\left(u(t-n_b)\right)\end{bmatrix}\in\mathbf{R}^{n_b\times m}。$$

为了识别系统并避免使用具有更高计算效率的过参数化方法[20-22]，使用递阶辨识原理[15-18]将式(5)所示的Hammerstein系统分解为如下所示的2个子系统：

$$S_1:\quad y(t)=\boldsymbol{\phi}^{\mathrm{T}}\left(\boldsymbol{c},p,t\right)\boldsymbol{\theta}+v(t),\tag{6}$$

$$S_2:\quad y_1(t)=\boldsymbol{\psi}^{\mathrm{T}}\left(\boldsymbol{b},p,t\right)\boldsymbol{\vartheta}+v(t)。\tag{7}$$

式中：

$$\boldsymbol{\vartheta}=\boldsymbol{c};$$

$$\boldsymbol{\theta}=\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix};$$

$$\boldsymbol{\phi}\left(\boldsymbol{c},p,t\right)=\begin{bmatrix}\boldsymbol{\varphi}(t)\\ \boldsymbol{F}(t)\boldsymbol{c}\end{bmatrix}\in\mathbf{R}^{n_a\times n_b};$$

$$\boldsymbol{\psi}\left(\boldsymbol{b},p,t\right)=\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{b}\in\mathbf{R}^{m};$$

$$y_1(t)=y(t)-\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{a}。$$

式中，p 为过去的水平数。定义加权数据矩阵和加权输出向量

$$\boldsymbol{Y}(p,t)=\begin{bmatrix}y(t)\\ \mu y(t-1)\\ \vdots\\ \lambda^{p-1}y(t-p+1)\end{bmatrix}\in\mathbf{R}^{p},\tag{8}$$

$$\boldsymbol{\Phi}_0(p,t)=\begin{bmatrix}\boldsymbol{\varphi}^{\mathrm{T}}(t)\\ \lambda\boldsymbol{\varphi}^{\mathrm{T}}(t-1)\\ \vdots\\ \lambda^{p-1}\boldsymbol{\varphi}^{\mathrm{T}}(t-p+1)\end{bmatrix}\in\mathbf{R}^{p\times n_a},\tag{9}$$

$$\boldsymbol{Y}_1(p,t)=\begin{bmatrix}y_1(t)\\ \lambda y_1(t-1)\\ \vdots\\ \lambda^{p-1}y_1(t-p+1)\end{bmatrix}=\boldsymbol{Y}(p,t)-\boldsymbol{\Phi}_0(p,t)\boldsymbol{a}\in\mathbf{R}^{p},\tag{10}$$

$$\boldsymbol{\Phi}\left(\boldsymbol{c},p,t\right)=\begin{bmatrix}\boldsymbol{\phi}^{\mathrm{T}}\left(\boldsymbol{c},p,t\right)\\ \lambda\boldsymbol{\phi}^{\mathrm{T}}\left(\boldsymbol{c},p,t-1\right)\\ \vdots\\ \lambda^{p-1}\boldsymbol{\phi}^{\mathrm{T}}\left(\boldsymbol{c},p,t-p+1\right)\end{bmatrix}\in\mathbf{R}^{p\times(n_a+n_b)},\tag{11}$$

$$\boldsymbol{\psi}(\boldsymbol{b},p,t)=\begin{bmatrix}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\\ \lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-1)\\ \vdots\\ \lambda^{p-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-p+1)\end{bmatrix}\in\mathbf{R}^{p\times m}。\tag{12}$$

式中，遗忘因子 $\lambda\in[0,1)$。

为了辨识未知的系统参数 $\boldsymbol{a}$、$\boldsymbol{b}$ 和 $\boldsymbol{c}$，定义如下的 2 个损失函数：

$$J_1(\boldsymbol{\theta})=\left\|\boldsymbol{Y}(p,t)-\boldsymbol{\Phi}(\boldsymbol{c},p,t)\boldsymbol{\theta}\right\|^2,\tag{13}$$

$$J_2(\boldsymbol{\vartheta})=\left\|\boldsymbol{Y}_1(p,t)-\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\vartheta}\right\|^2。\tag{14}$$

令 J_1J_1 和 J_2 对 $\boldsymbol{\theta}$ 和 $\boldsymbol{\vartheta}$ 的导数为 0，有

$$\frac{\partial J_1(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=2\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\left[\boldsymbol{Y}(p,t)-\boldsymbol{\Phi}(\boldsymbol{c},p,t)\boldsymbol{\theta}\right]=0,\tag{15}$$

$$\frac{\partial J_2(\boldsymbol{\vartheta})}{\partial\boldsymbol{\vartheta}}=2\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\left[\boldsymbol{Y}_1(p,t)-\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\theta}\right]=0。\tag{16}$$

$\boldsymbol{\theta}$ 和 $\boldsymbol{\vartheta}$ 的最小二乘估计值由下式给出：

$$\hat{\boldsymbol{\theta}}(t)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{\Phi}(\boldsymbol{c},p,t)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{Y}(p,t),\tag{17}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{Y}_1(p,t)。\tag{18}$$

类似于(17)和(18)，有

$$\hat{\boldsymbol{\theta}}(t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p,t-1)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t-1)\boldsymbol{Y}(p,t-1),\tag{19}$$

$$\hat{\boldsymbol{\vartheta}}(t-1)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},pt-1)\boldsymbol{\psi}(\boldsymbol{b},p,t-1)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-1)\boldsymbol{Y}_1(p,t-1)。\tag{20}$$

类似于(8)~(12)，定义

$$\boldsymbol{\Phi}(\boldsymbol{c},p,t)=\begin{bmatrix}\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\\ \lambda\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\end{bmatrix},\tag{21}$$

$$\boldsymbol{Y}(p,t)=\begin{bmatrix}y(t)\\ \lambda\boldsymbol{Y}(p-1,t-1)\end{bmatrix},\tag{22}$$

$$\boldsymbol{\Phi}(\boldsymbol{c},p,t-1)=\begin{bmatrix}\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\\ \lambda^p\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)\end{bmatrix},\tag{23}$$

$$\boldsymbol{Y}(p,t-1)=\begin{bmatrix}\boldsymbol{Y}(p-1,t-1)\\ \lambda^p y(t-p)\end{bmatrix},\tag{24}$$

$$\boldsymbol{\psi}(\boldsymbol{b},p,t)=\begin{bmatrix}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\\ \lambda\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\end{bmatrix},\tag{25}$$

$$\boldsymbol{Y}_1(p,t)=\begin{bmatrix}y_1(t)\\ \lambda\boldsymbol{Y}_1(p-1,t-1)\end{bmatrix},\tag{26}$$

$$\boldsymbol{\psi}(\boldsymbol{b},p,t-1)=\begin{bmatrix}\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\\ \lambda^p\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)\end{bmatrix},\tag{27}$$

$$\boldsymbol{Y}_1(p,t-1)=\begin{bmatrix}\boldsymbol{Y}_1(p-1,t-1)\\ \lambda^p y_1(t-p)\end{bmatrix}。\tag{28}$$

定义矩阵

$$\boldsymbol{P}(t)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{\Phi}(\boldsymbol{c},p,t)\right]^{-1},\tag{29}$$

$$\boldsymbol{P}_{\alpha}(t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\right]^{-1},\tag{30}$$

$$\boldsymbol{\alpha}(t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t)\boldsymbol{Y}(p-1,t-1),\tag{31}$$

$$\boldsymbol{P}_{1}(t)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\right]^{-1},\tag{32}$$

$$\boldsymbol{P}_{1\alpha}(t-1)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\right]^{-1},\tag{33}$$

$$\boldsymbol{\alpha}_{1}(t-1)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t)\boldsymbol{Y}_{1}(p-1,t-1)。\tag{34}$$

由式(29)~(34)得

$$\boldsymbol{P}^{-1}(t)=\left[\boldsymbol{\phi}(\boldsymbol{c},p,t)\ \lambda\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\right]\begin{bmatrix}\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\\ \lambda\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\end{bmatrix}=\lambda\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)=\lambda\boldsymbol{P}_{\alpha}^{-1}(t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t),\tag{35}$$

$$\boldsymbol{P}^{-1}(t-1)=\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p,t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\ \lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)\right]\begin{bmatrix}s\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\\ \lambda^{p}\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)\end{bmatrix}=\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)+\lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)=\boldsymbol{P}_{\alpha}^{-1}(t-1)+\lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p),\tag{36}$$

$$\boldsymbol{\alpha}(t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p-1,t-1)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{Y}(p-1,t-1)=\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{Y}(p-1,t-1),\tag{37}$$

$$\boldsymbol{P}_{1}^{-1}(t)=\left[\boldsymbol{\psi}(\boldsymbol{b},p,t)\ \lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\right]\begin{bmatrix}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\\ \lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\end{bmatrix}=\lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)=\lambda\boldsymbol{P}_{1\alpha}^{-1}(t-1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t),\tag{38}$$

$$\boldsymbol{P}_{1}^{-1}(t-1)=\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-1)\boldsymbol{\psi}(\boldsymbol{b},p,t-1)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\ \lambda^{p}\boldsymbol{\psi}(\boldsymbol{b},t-p)\right]\begin{bmatrix}s\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\\ \lambda^{p}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)\end{bmatrix}=\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)+\lambda^{p}\boldsymbol{\psi}(\boldsymbol{b},t-p)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)=\boldsymbol{P}_{1\alpha}^{-1}(t-1)+\lambda^{p}\boldsymbol{\psi}(\boldsymbol{b},t-p)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p),\tag{39}$$

$$\boldsymbol{\alpha}_{1}(t-1):=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{\psi}(\boldsymbol{b},p-1,t-1)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{Y}_{1}(p-1,t-1)=\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{Y}_{1}(p-1,t-1)。\tag{40}$$

把式(29)~(34)带入式(19)~(20)中，有

$$\hat{\boldsymbol{\theta}}(t)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{\Phi}(\boldsymbol{c},p,t)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{Y}(p,t)=\boldsymbol{P}(t)\left[\boldsymbol{\phi}(\boldsymbol{c},p,t)\ \lambda\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\right]\begin{bmatrix}y(t)\\ \lambda\boldsymbol{Y}(p-1,t-1)\end{bmatrix}=\boldsymbol{P}(t)\left[\lambda\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{Y}(p-1,t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)y(t)\right]=\boldsymbol{P}(t)\left[\lambda\boldsymbol{P}_{\alpha}^{-1}(t-1)\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{Y}(p-1,t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)y(t)\right]=\boldsymbol{P}(t)\left[\lambda\boldsymbol{P}_{\alpha}^{-1}(t-1)\boldsymbol{\alpha}(t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)y(t)\right]=\lambda\boldsymbol{P}(t)\boldsymbol{P}_{\alpha}^{-1}(t-1)\boldsymbol{\alpha}(t-1)+\boldsymbol{P}(t)\boldsymbol{\phi}(\boldsymbol{c},p,t)y(t)=\left[\boldsymbol{I}-\boldsymbol{P}(t)\boldsymbol{\phi}(\boldsymbol{c},p,t)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\right]\boldsymbol{\alpha}(t-1)+\boldsymbol{P}(t)\boldsymbol{\phi}(\boldsymbol{c},p,t)y(t)=\boldsymbol{\alpha}(t-1)+\boldsymbol{P}(t)\boldsymbol{\phi}(\boldsymbol{c},p,t)\left[y(t)-\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{\alpha}(t-1)\right],\tag{41}$$

$$\hat{\boldsymbol{\theta}}(t-1)=\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t-1)\boldsymbol{\Phi}(\boldsymbol{c},p,t-1)\right]^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p,t-1)\boldsymbol{Y}(p,t-1)=\boldsymbol{P}(t-1)\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\ \lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)\right]\begin{bmatrix}\boldsymbol{Y}(p-1,t-1)\\ \lambda^{p}y(t-p)\end{bmatrix}=\boldsymbol{P}(t-1)\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{c},p-1,t-1)\boldsymbol{Y}(p-1,t-1)+\lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)y(t-p)\right]=\boldsymbol{P}(t-1)\left[\boldsymbol{P}_{\alpha}^{-1}(t-1)\boldsymbol{\alpha}(t-1)+\lambda^{p}\boldsymbol{\phi}(\boldsymbol{c},t-p)y(t-p)\right],\tag{42}$$

$$\boldsymbol{\alpha}(t-1)=\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{P}^{-1}(t-1)\hat{\boldsymbol{\theta}}(t-1)-\lambda^{p}\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\boldsymbol{c},t-p)y(t-p)=\left[\boldsymbol{I}-\lambda^{p}\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\boldsymbol{c},t-p)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)\right]\hat{\boldsymbol{\theta}}(t-1)-\lambda^{p}\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\boldsymbol{c},t-p)y(t-p)=\hat{\boldsymbol{\theta}}(t-1)-\lambda^{p}\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\boldsymbol{c},t-p)\left[y(t-p)+\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)\hat{\boldsymbol{\theta}}(t-1)\right],\tag{43}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{Y}_1(p,t)=\boldsymbol{P}_1(t)\left[\boldsymbol{\psi}(\boldsymbol{b},p,t)\ \lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\right]\begin{bmatrix}y_1(t)\\ \lambda\boldsymbol{Y}_1(p-1,t-1)\end{bmatrix}=$$
$$\boldsymbol{P}_1(t)\left[\lambda\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{Y}_1(p-1,t-1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)y_1(t)\right]=\boldsymbol{P}_1(t)\left[\lambda\boldsymbol{P}_{1\alpha}^{-1}(t-1)\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{Y}_1(p-1,t-$$
$$1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)y_1(t)\right]=\boldsymbol{P}_1(t)\left[\lambda\boldsymbol{P}_{1\alpha}^{-1}(t-1)\boldsymbol{\alpha}_1(t-1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)y_1(t)\right]=\lambda\boldsymbol{P}_1(t)\boldsymbol{P}_{1\alpha}^{-1}(t-1)\boldsymbol{\alpha}_1(t-1)+$$
$$\boldsymbol{P}_1(t)\boldsymbol{\psi}(\boldsymbol{b},p,t)y_1(t)=\left[\boldsymbol{I}-\boldsymbol{P}_1(t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\right]\boldsymbol{\alpha}_1(t-1)+\boldsymbol{P}_1(t)\boldsymbol{\psi}(\boldsymbol{b},p,t)y_1(t)=\boldsymbol{\alpha}_1(t-1)+$$
$$\boldsymbol{P}_1(t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\left[y_1(t)-\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{\alpha}_1(t-1)\right],\tag{44}$$

$$\hat{\boldsymbol{\vartheta}}(t-1)=\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-1)\boldsymbol{\psi}(\boldsymbol{b},p,t-1)\right]^{-1}\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t-1)\boldsymbol{Y}_1(p,t-1)=\boldsymbol{P}_1(t-1)\times$$
$$\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\ \lambda^p\boldsymbol{\psi}(\boldsymbol{b},t-p)\right]\begin{bmatrix}\boldsymbol{Y}_1(p-1,t-1)\\ \lambda^p y_1(t-p)\end{bmatrix}=\boldsymbol{P}_1(t-1)\left[\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p-1,t-1)\boldsymbol{Y}_1(p-1,t-1)+\lambda^p\boldsymbol{\psi}(\boldsymbol{b},t-p)y_1(t-p)\right]=$$
$$\boldsymbol{P}_1(t-1)\left[\boldsymbol{P}_{1\alpha}^{-1}(t-1)\boldsymbol{\alpha}_1(t-1)+\lambda^p\boldsymbol{\psi}(\boldsymbol{b},t-p)y_1(t-p)\right],\tag{45}$$

$$\boldsymbol{\alpha}_1(t-1)=\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{P}_1^{-1}(t-1)\hat{\boldsymbol{\vartheta}}(t-1)-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\boldsymbol{b},t-p)y_1(t-p)=\left[\boldsymbol{I}-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\boldsymbol{b},t-p)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-$$
$$p)\right]\hat{\boldsymbol{\vartheta}}(t-1)-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\boldsymbol{b},t-p)y_1(t-p)=\hat{\boldsymbol{\vartheta}}(t-1)-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\boldsymbol{b},t-p)\left[y_1(t-p)+\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)\hat{\boldsymbol{\vartheta}}(t-1)\right]。\tag{46}$$

通过上述等式,可以得到如下 FF-FDW-RLS 算法:

$$\hat{\boldsymbol{\theta}}(t)=\boldsymbol{\alpha}(t-1)+\boldsymbol{P}(t)\boldsymbol{\phi}(\boldsymbol{c},p,t)\left[y(t)-\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t)\boldsymbol{\alpha}(t-1)\right],\tag{47}$$
$$\boldsymbol{P}^{-1}(t)=\lambda\boldsymbol{P}_{\alpha}^{-1}(t-1)+\boldsymbol{\phi}(\boldsymbol{c},p,t)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},p,t),\tag{48}$$
$$\boldsymbol{\alpha}(t-1)=\hat{\boldsymbol{\theta}}(t-1)-\lambda^p\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\boldsymbol{c},t-p)\left[y(t-p)+\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p)\hat{\boldsymbol{\theta}}(t-1)\right],\tag{49}$$
$$\boldsymbol{P}_{\alpha}^{-1}(t-1)=\boldsymbol{P}^{-1}(t-1)-\lambda^p\boldsymbol{\phi}(\boldsymbol{c},t-p)\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{c},t-p),\tag{50}$$
$$\hat{\boldsymbol{\vartheta}}(t)=\boldsymbol{\alpha}_1(t-1)+\boldsymbol{P}_1(t)\boldsymbol{\psi}(\boldsymbol{b},p,t)\left[y_1(t)-\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t)\boldsymbol{\alpha}_1(t-1)\right],\tag{51}$$
$$\boldsymbol{P}_1^{-1}(t)=\lambda\boldsymbol{P}_{1\alpha}^{-1}(t-1)+\boldsymbol{\psi}(\boldsymbol{b},p,t)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},p,t),\tag{52}$$
$$\boldsymbol{\alpha}_1(t-1)=\hat{\boldsymbol{\vartheta}}(t-1)-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\boldsymbol{b},t-p)\left[y_1(t-p)+\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)\hat{\boldsymbol{\vartheta}}(t-1)\right],\tag{53}$$
$$\boldsymbol{P}_{1\alpha}^{-1}(t-1)=\boldsymbol{P}_1^{-1}(t-1)-\lambda^p\boldsymbol{\psi}(\boldsymbol{b},t-p)\boldsymbol{\psi}^{\mathrm{T}}(\boldsymbol{b},t-p)。\tag{54}$$

由于式(47)~(54)中包含了未知参数 $\boldsymbol{a}$、$\boldsymbol{b}$、$\boldsymbol{c}$,这些未知参数是无法使用的。用估计值 $\hat{\boldsymbol{a}}_{(t-1)}$、$\hat{\boldsymbol{b}}_{(t-1)}$、$\hat{\boldsymbol{c}}_{(t-1)}$来代替真实值[23]:

$$\hat{\boldsymbol{\vartheta}}(t)=\boldsymbol{\alpha}_1(t-1)+\boldsymbol{P}_1(t)\boldsymbol{\psi}(\hat{\boldsymbol{b}}_{(t-1)},p,t)\left[y_1(t)-\boldsymbol{\psi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{(t-1)},p,t)\boldsymbol{\alpha}_1(t-1)\right],\tag{55}$$
$$\boldsymbol{P}_1^{-1}(t)=\lambda\boldsymbol{P}_{1\alpha}^{-1}(t-1)+\boldsymbol{\psi}(\hat{\boldsymbol{b}}_{(t-1)},p,t)\boldsymbol{\psi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{(t-1)},p,t),\tag{56}$$
$$\boldsymbol{\alpha}_1(t-1)=\hat{\boldsymbol{\vartheta}}(t-1)-\lambda^p\boldsymbol{P}_{1\alpha}(t-1)\boldsymbol{\psi}(\hat{\boldsymbol{b}}_{(t-2)},t-p)\left[y_1(t-p)+\boldsymbol{\psi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{(t-2)},t-p)\hat{\boldsymbol{\vartheta}}(t-1)\right],\tag{57}$$
$$\boldsymbol{P}_{1\alpha}^{-1}(t-1)=\boldsymbol{P}_1^{-1}(t-1)-\lambda^p\boldsymbol{\psi}(\hat{\boldsymbol{b}}_{(t-2)},t-p)\boldsymbol{\psi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{(t-2)},t-p),\tag{58}$$
$$\hat{\boldsymbol{\theta}}(t)=\boldsymbol{\alpha}(t-1)+\boldsymbol{P}(t)\boldsymbol{\phi}(\hat{\boldsymbol{c}}_{(t-1)},p,t)\left[y(t)-\boldsymbol{\phi}^{\mathrm{T}}(\hat{\boldsymbol{c}}_{(t-1)},p,t)\boldsymbol{\alpha}(t-1)\right],\tag{59}$$
$$\boldsymbol{P}^{-1}(t)=\lambda\boldsymbol{P}_{\alpha}^{-1}(t-1)+\boldsymbol{\phi}(\hat{\boldsymbol{c}}_{(t-1)},p,t)\boldsymbol{\phi}^{\mathrm{T}}(\hat{\boldsymbol{c}}_{(t-1)},p,t),\tag{60}$$
$$\boldsymbol{\alpha}(t-1)=\hat{\boldsymbol{\theta}}(t-1)-\lambda^p\boldsymbol{P}_{\alpha}(t-1)\boldsymbol{\phi}(\hat{\boldsymbol{c}}_{(t-2)},t-p)\left[y(t-p)+\boldsymbol{\phi}^{\mathrm{T}}(\hat{\boldsymbol{c}}_{(t-2)},t-p)\hat{\boldsymbol{\theta}}(t-1)\right],\tag{61}$$
$$\boldsymbol{P}_{\alpha}^{-1}(t-1)=\boldsymbol{P}^{-1}(t-1)-\lambda^p\boldsymbol{\phi}(\hat{\boldsymbol{c}}_{(t-2)},t-p)\boldsymbol{\phi}^{\mathrm{T}}(\hat{\boldsymbol{c}}_{(t-2)},t-p)。\tag{62}$$

为了初始化 FF-FDW-RLS 算法,取 $\hat{\boldsymbol{\theta}}(0)=\vec{0}$,$\hat{\boldsymbol{\vartheta}}(0)=\vec{0}$或一些小的实数向量,例如,$\hat{\boldsymbol{\theta}}(0)=10^{-6}\boldsymbol{i}_n$,$\hat{\boldsymbol{\vartheta}}(0)=10^{-6}\boldsymbol{i}_n$,其中 $\boldsymbol{i}_n$ 是元素全为 1 的 n 维列向量;$\boldsymbol{P}(0)=p_0\boldsymbol{I}$,$\boldsymbol{P}_{\alpha}(0)=p_0\boldsymbol{I}$,$\boldsymbol{P}_1(0)=p_0\boldsymbol{I}$,$\boldsymbol{P}_{1\alpha}(0)=p_0\boldsymbol{I}$,其中 $\boldsymbol{I}$ 是单位矩阵,p_0 是很大的正数,例如 $p_0=10^6$。

随着 t 增加,用 FF-FDW-RLS 算法计算 $\hat{\boldsymbol{\theta}}(t)$ 和 $\hat{\boldsymbol{\vartheta}}(t)$ 的步骤如下。

1)采集输入输出数据 $\{u(t),y(t):t=1,2,\cdots,L\}$,选择数据长度 p 和遗忘因子 λ,得到 $\boldsymbol{\phi}(\hat{\boldsymbol{c}}_{(t-1)},p,t)$,

$\psi\left(\hat{b}_{(t-1)},p,t\right)$。

2）通过式(55)和(59)，计算$\hat{\theta}(t)$和$\hat{\vartheta}(t)$。

3）如果$t=L$，则终止程序并得到参数估计值；否则，递增1并转到步骤3。

3 仿真研究

考虑参考文献[24]中的Hammerstein系统如下：

$$A(z)y(t)=B(z)\bar{u}(t)+v(t),$$
$$\bar{u}(t)=0.8\sin(u(t))+0.6\cos(u^2(t)),$$
$$A(z):=1-1.35z^{-1}+0.65z^{-2},$$
$$B(z):=1.68z^{-1}+1.32z^{-2},$$
$$\boldsymbol{\Theta}=[\boldsymbol{\theta}^{\mathrm{T}}\ \boldsymbol{\vartheta}]^{\mathrm{T}}。$$

其中输入采用均值为0、方差为1的白噪声，噪声是均值为0、方差为0.01或0.04的白噪声。取p=10，遗忘因子λ=0.98。用所提出的算法和H-MISG算法[24]（p=10，λ=0.98）来估计这个系统的参数。图1和图2分别表示在噪声方差为0.01和0.04的情况下，参数估计的误差$\delta=\left\|\hat{\boldsymbol{\Theta}}(t)-\boldsymbol{\Theta}\right\|/\|\boldsymbol{\Theta}\|$与$t$的关系。可以看出，与参考文献[24]中提出的H-MISG算法相比，本研究中的方法可以快速跟踪估计的参数，可以提高识别精度和收敛速度。

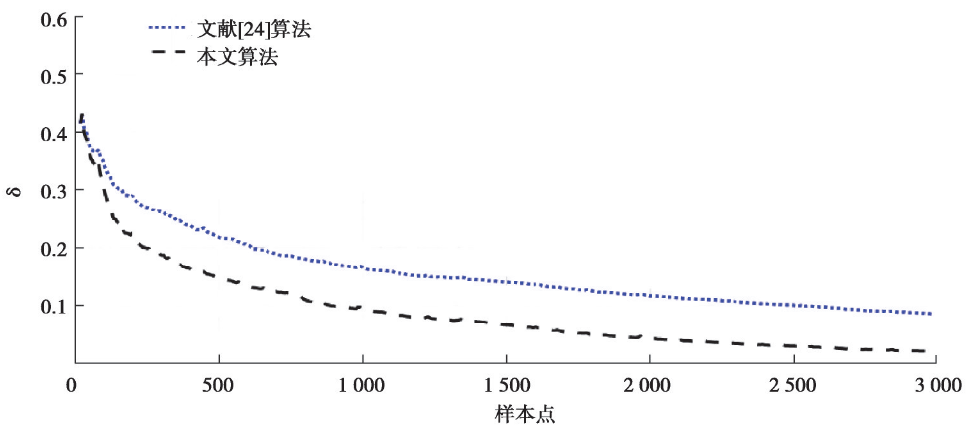

图2 噪声方差为0.01时，参数估计误差$\delta=\left\|\hat{\boldsymbol{\Theta}}(t)-\boldsymbol{\Theta}\right\|/\|\boldsymbol{\Theta}\|$与$t$的关系

Fig. 2 Parameter estimation errors of $\delta=\left\|\hat{\boldsymbol{\Theta}}(t)-\boldsymbol{\Theta}\right\|/\|\boldsymbol{\Theta}\|$ versus t for noise variance of 0.01

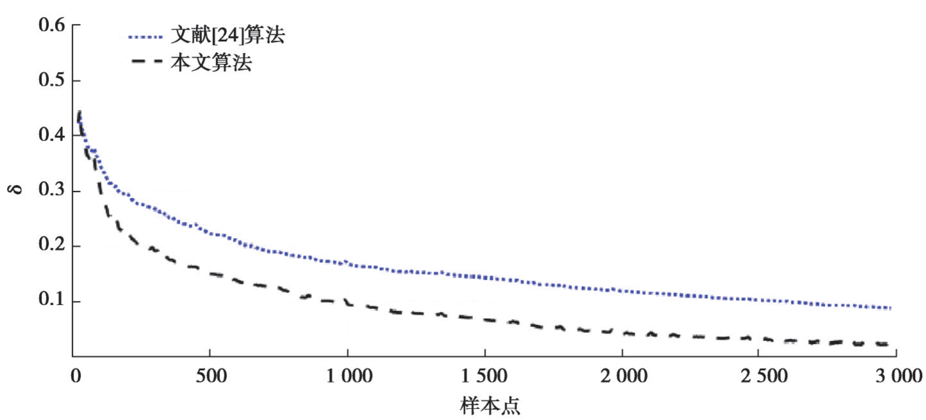

图3 噪声方差为0.04时，参数估计误差$\delta=\left\|\hat{\boldsymbol{\Theta}}(t)-\boldsymbol{\Theta}\right\|/\|\boldsymbol{\Theta}\|$与$t$的关系

Fig. 3 The parameter estimation errors of $\delta=\left\|\hat{\boldsymbol{\Theta}}(t)-\boldsymbol{\Theta}\right\|/\|\boldsymbol{\Theta}\|$ versus t for noise variances of 0.04

4 结　论

提出了一种基于遗忘因子的分解递归有限数据窗口最小二乘法来识别Hammerstein系统，该方法同时保留了FF-LS和FDW-LS的优点，能够快速跟踪参数实现对Hammerstein系统的精确有效辨识，通过仿真实验验证了本文方法的有效性，和现有方法相比，通过引入窗口遗忘因子得到了较快的跟踪效果。

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